Online Offering Strategies for Storage-Assisted Renewable Power Producer in Hour-Ahead Market

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Abstract

A promising approach to hedge against the inherent uncertainty of renewable generation is to equip the renewable plants with energy storage systems. This paper focuses on designing profit maximization offering strategies, *i.e.*, the strategies that determine the offering price and volume, for a storage-assisted renewable power producer that participates in hour-ahead electricity market. Designing the strategies is challenging since (i) the underlying problem is coupled across time due to the evolution of the storage level, and (ii) inputs to the problem including the renewable output and market clearing price are unknown when submitting offers. Following the competitive online algorithm design approach, we first study a basic setting where the renewable output and the clearing price are known for the next hour. We propose sOffer, a simple online offering strategy that achieves the best possible competitive ratio of $O(\log \theta)$, where θ is the ratio between the maximum and the minimum clearing prices. Then, we consider the case where the clearing price is unknown. By exploiting the idea of submitting multiple offers to combat price uncertainty, we propose mOffer, and demonstrate that the competitive ratio of mOffer converges to that of sOffer as the number of offers grows. Finally, we extend our approach to the scenario where the renewable output has forecasting error. We propose gOffer as the generalized offering strategy and characterize its competitive ratio as a function of the forecasting error. Our trace-driven experiments demonstrate that our algorithms achieve performance close to the offline optimal and outperform a baseline alternative significantly.

1   Introduction

Nowadays, renewable power producers, such as wind farms and solar plants, are being rapidly integrated to the power system. In 2015, investment on renewables sets a record of 296 billion dollars, more than double the amount for fossil fuels [9]. Renewables are attractive in that they are clean, free (except capital and maintenance cost), and inexhaustible. Integration of renewables into the power system and particularly in the electricity market, however, is challenging since their generation is uncontrollable, intermittent, and unpredictable. A promising approach to facilitate the renewable integration and hedge against the uncertainty, is to equip the renewable plants with the giant energy storage systems [14, 15]. Some examples are the storage stations at Southern California (with capacity of 40MWh), South Korea (16MWh), and Germany (15MWh) [1].

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In several existing electricity markets, renewable power producers receive guaranteed feed-in tariffs, *e.g.*, "take-all-wind" policy in California's electricity market [3]. "take-all-wind" policy guarantees that the market absorbs the entire renewable supply at favorable fixed prices. This policy is feasible since the current market share of the renewables is not significant, at most 19.2%, on average around the world, in 2015 [9].

The extra-market treatment policy, however, is not viable in future. Recently, the deployment cost of the renewables is dropping rapidly, *e.g.*, the price of solar panel has fallen by 80% from 2008 to 2016 [2]. This reduction in price along with the environmental concerns pushes rapid penetration of the renewables, *e.g.*, in Denmark, the plan is to achieve 50% and 100% renewable generation in 2020 and 2050, respectively [7]. Hence, in eventual market with the considerable renewables' market share, it is inevitable to treat renewable producers the same as other traditional generation companies [20].

Several electricity markets such as NYISO [5], CAISO [3], and Nord Pool [6] operate in a multi-settlement manner and settle transactions at multiple timescales, *i.e.*, day-ahead, hour-ahead [17, 19], and real-time. Considering the uncertainty of renewable output, srGENCO tends to participate in short-term market, specifically hour-ahead market, without suffering profit reduction caused by long-term forecasting errors [20]. In reality, CAISO's Participating Intermittent Resource Program (PIRP [3]) is already requiring wind power plants to bid into their hour-ahead market. In hour-ahead market operation, the generation companies including renewable producers submit their *offers* (including offering price and offering volume) for selling the electricity in the next hour (see Sec. 2.1 for more details on the hour-ahead market operation). This work focuses on designing offering strategies, *i.e.*, the strategies that determine the offering price and volume, for Storage-assisted Renewable GENeration COmpany (srGENCO) that participates in hour-ahead market.

As depicted in Fig. 1, we consider a scenario in which an srGENCO, like other traditional generation companies, participates in hour-ahead market by (1) submitting the offer. After receiving the offers, (2) the market operator matches the offers with the bids from the demand-side and announces a market clearing price. If the offering price of srGENCO is less than the clearing price, (3) its offering volume is considered as the *commitment* to the market for the next hour. In turn, srGENCO is paid according to the clearing price.
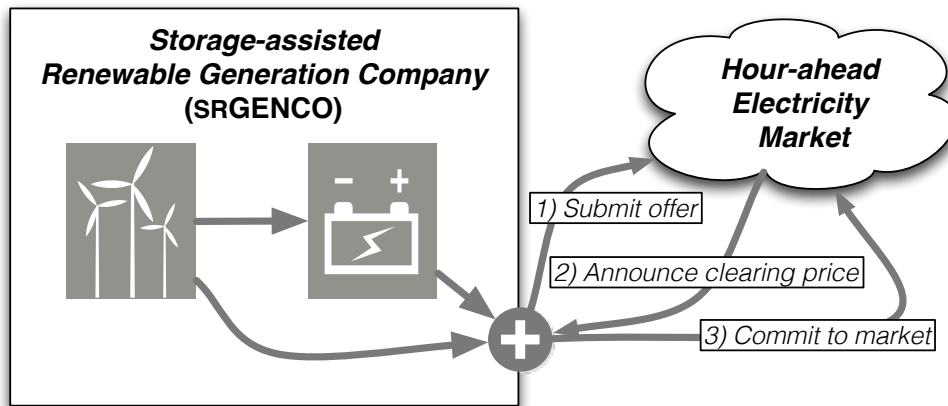

Figure 1: The scenario

Finding profit maximization offering strategy for a renewable producer without storage, is nontrivial

due to the inherent uncertainty of the renewables and dynamics in the market clearing price. In the presence of storage, the offering strategy is even more challenging because of the additional design space enabled by the storage. More specifically, at first, srGENCO can use the storage to compensate for the slots that the renewable output cannot fulfill the commitment. In addition, the storage can potentially offer another economic advantage. That is, it can shift the energy through absorbing the renewable output during low price periods, and then discharge it during the high price periods. In this way, designing profit maximization offering strategy in the presence of storage comes with wider design space than those without the storage and potentially can bring more profit for srGENCO.

To design the offering strategy, this paper casts an optimization problem with the objective of maximizing the long-term profit of srGENCO subject to the storage capacity constraints. The inputs to the problem, *i.e.*, the renewable output and the clearing price, however, are unknown for srGENCO when submitting offer. This emphasizes the need for online solution design which is challenging, since the problem is coupled across time due to the evolution of the storage. We note that some similar problems have been studied in literature using offline [13, 18] and stochastic optimization approaches [19, 20]. We refer to Sec. 2.4 for details.

By introducing the system model and problem formulation in Sec. 2, this work tackles the profit maximization offering strategy problem following online competitive algorithm design [12] and makes the following contributions:

1. In Sec. 3, we study a basic setting where the exact values of renewable output and the clearing price for the next slot are known to srGENCO before submitting the offer. Even under this idealized setting, solving the problem is still challenging since the input to problem is still unknown for future slots beyond the next one. We propose sOffer, a simple online offering strategy, in which the offering volume is calculated through a piecewise exponential/constant function of the renewable output and the current storage level, as well as the clearing price of the next slot. Our analysis demonstrates that sOffer achieves the best possible competitive ratio of $O(\log \theta)$, where θ is the ratio between the maximum and minimum clearing prices.

2. In Sec. 4.1, we study the case where the clearing price is unknown and propose mOffer. In mOffer, srGENCO submits multiple offers, each of which conveys a portion of the total offering volume, at different offering prices. Our analysis shows that the competitive ratio of mOffer converges to the ratio of sOffer as the number of offers grows. Moreover, in Sec. 4.2, our approach is extended to the case where srGENCO knows renewable output with forecasting error. We propose gOffer as the generalized offering strategy and characterize the competitive ratio as a function of the maximum forecasting error.

3. In Sec. 5, by extensive numerical experiments based on real-world traces, we show that our online offering strategies can achieve satisfactory performance as compared to the offline optimum. In addition, gOffer with 10% forecasting error improves the profit of srGENCO by 15% as compared to the baseline scenario without storage. As notable observations, our experiments demonstrate that when the market clearing price is unknown, submitting 3 offers is sufficient to achieve almost the same performance as the case that the market price is known. Moreover, forecasting error of more than 20% significantly degrades the performance results. In summary, our observations demonstrate that, while the uncertainty in market price can be effectively handled by multiple offer submissions, accurate short-term renewable forecasting is vital for srGENCO to obtain a desired profit.

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Table 1: Summary of key notations

Notation	Description
t	Index of one-hour time slot
T	The number of time slots, $T \geq 0$
$\mathcal{T}$	Set $\mathcal{T} = \{1, 2, \dots, T\}$
$p(t)$	Market clearing price at t, $p_{\min} \leq p(t) \leq p_{\max}$
θ	The ratio between the maximum and the minimum clearing prices, $\theta = p_{\max}/p_{\min}$
$u(t)$	The renewable output of srGENCO at t
C	The capacity of storage system
ρ_c	The maximum charging rate of storage system
ρ_d	The maximum discharging rate of storage system
$z(t)$	The storage level at the beginning of t, see Eq. (2)
$\hat{p}(t)$	**optimization variable**, offering price of srGENCO at t
$\hat{x}(t)$	**optimization variable**, offering volume of srGENCO at t
$x(t)$	Commitment volume of srGENCO at t, see Eq. (1)
$y(t)$	Over-commitment volume of srGENCO at t, see Eq. (5)
$R(t)$	The net profit of srGENCO at t, see Eq. (6)

We conclude the paper in Sec. 6. Due to space limitation, all proofs are included in our technical report [10].

2 Model and Problem Formulation

2.1 Hour-Ahead Electricity Market

The hour-ahead market operates in hourly basis[1] and srGENCO along with other generation companies submits its offer, including the offering price and the offering volume (see Sec. 2.2.2 for details), for the forthcoming hour shortly before the operation time. The market operator (usually

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[1]We emphasize that our model works in short-term market in which the offers are submitted *before* actual market operation in hourly or even shorter scales, *e.g.*, 15 or 5 minutes ahead. For real examples we refer to California ISO [3] and Nord Pool Markets [6]. For a survey of electricity markets, we refer to [17].

known as independent system operator, ISO) matches the offers collected from the generation companies with the received bids from the demand-side, *e.g.*, utility companies. Then, using a well-established double auction mechanism [21] it determines the market clearing price for the next hour. The generation companies with the offering prices less than the clearing price are successful and the offering volume of electricity is considered as their *commitment* to be sold on the market at the clearing price. Thus, successful offers sell at price at least as high as what they offered. All the remaining offers fail since their offering prices are greater than the clearing price.

Formally, we consider a time-slotted model, such that the time horizon T is chopped into multiple slots with equal length, *e.g.*, 1 hour, each of which is indexed by t. Shortly before slot t, srGENCO along with other participants submits its offer, for the next slot. The ISO determines the clearing price $p(t)$ shortly after the participants submit their offers and bids. We assume $p_{\min} \leq p(t) \leq p_{\max}$, and parameter θ is defined as the ratio between the maximum and the minimum clearing prices, *i.e.*, $\theta = p_{\max}/p_{\min}$. We later use θ to analyze our algorithms.

2.2 The properties of srGENCO

There is a Storage-assisted Renewable GENeration COmpany (srGENCO) that produces electricity from the renewable sources such as wind farm or solar plant. At the same time srGENCO is equipped with the storage systems to store the electricity for future commitment with potentially higher price. On the other hand, the storage could be discharged to compensate for the shortage of renewable output when the commitment to the market is beyond the renewable output.

2.2.1 Renewable Output

The renewable output of srGENCO at slot t is denoted by $u(t)$ and we do not rely on any specific stochastic model of $u(t)$. In general, we assume that srGENCO does not know the exact amount of $u(t)$ when submitting the offer. Note that $u(t)$ could be (i) directly committed to the market, (ii) be committed to the market partially while the residual is stored on the storage, or (iii) entirely be stored on the storage for future usage.

2.2.2 Offering Strategy

By offering strategy we mean the way that srGENCO determines its offer that includes:

(i) **Offering price** denoted as $\hat{p}(t) \in [p_{\min}, p_{\max}]$, *i.e.*, the minimum price at which srGENCO desires to commit electricity to the market.

(ii) **Offering volume** denoted as $\hat{x}(t) \geq 0$, *i.e.*, the amount of electricity in MWh at which srGENCO offers to the market at slot t.[2]

We distinguish between the offering volume and the *commitment volume*. After the clearing price $p(t)$ is revealed, srGENCO's offer may or may not be successful. If the offer is successful the offering volume is considered as the commitment volume. Otherwise, there would be no commitment. More

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[2]Note that there is no upper bound for $\hat{x}(t)$ because we assume that the market is big enough to absorb the offering volume of srGENCO entirely.

specifically, we define $x(t)$ as the commitment volume of SRGENCO at slot t as

$$x(t) = \begin{cases} \hat{x}(t) & \text{if } p(t) \geq \hat{p}(t), \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The goal of this study is to design an offering strategy for SRGENCO to submit both offering price $\hat{p}(t)$ and offering volume $\hat{x}(t)$ so as to maximize its long-term profit.

2.2.3 Storage Model

We denote the maximum capacity of storage system of SRGENCO by C and let ρ_c and ρ_d be its maximum charging and discharge rates, respectively. In addition, let $z(t) \in [0, C]$ be the storage level at the *beginning* of slot t. Given the renewable output $u(t)$ and the commitment volume $x(t)$, the evolution of the storage level of SRGENCO is given by

$$z(t + 1) = \Big[z(t) + x_c(t) - x_d(t) \Big]_{\mathcal{C}}, \tag{2}$$

where

$$x_c(t) = \min \Big\{ \rho_c, \big[u(t) - x(t) \big]^+ \Big\}, \tag{3}$$

is the charging amount of the storage at slot t,

$$x_d(t) = \min \Big\{ \rho_d, \big[x(t) - u(t) \big]^+ \Big\}, \tag{4}$$

and $x_d(t)$ is the discharging amount of the storage at slot t. Moreover, $[.]^+$ and $[.]_{\mathcal{C}}$ define the projections onto the positive orthant and set $\mathcal{C} = [0, C]$, respectively. Since SRGENCO is empowered by the storage, the commitment volume $x(t)$ might be either greater, less, or equal to the renewable output $u(t)$. The evolution of the storage for each case is as follows:

(i) $u(t) = x(t)$: in this case the entire renewable output is committed to the market and there is no change on the storage level, *i.e.*, $z(t + 1) = z(t)$.

(ii) $u(t) > x(t)$: in this case, $u(t) - x(t) > 0$ represents the amount of the surplus in the renewable output. Ideally, this surplus must be charged into the storage for the forthcoming commitments. However, because of the charging rate ρ_c it may not be possible, which is indeed captured in Eq. (3).

(iii) $u(t) < x(t)$: in this case not only the entire renewable output is committed, but also the storage should contribute in fulfilling the residual commitment, *i.e.*, $x(t) - u(t) > 0$. Again, given the storage level $z(t)$, and the discharge rate ρ_d, the residual commitment may not be satisfied.

2.2.4 Over-Commitment

Recall that SRGENCO does not know the exact renewable output $u(t)$ when submitting the offer, hence, it may fail to fulfill its commitment, which we refer to as the over-commitment. Let us denote $y(t)$ as the over-commitment volume at t expressed as

$$y(t) = \Big[x(t) - \Big(u(t) + \min \big\{ z(t), \rho_d \big\} \Big) \Big]^+. \tag{5}$$

Note that the maximum amount that SRGENCO can provide to the market in operation time t is the aggregation of the renewable output $u(t)$ and the maximum amount that could be discharged

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from the storage, *i.e.*, $\min\{z(t), \rho_d\}$. Since $u(t)$ is unknown to SRGENCO when submitting the offer, the commitment volume $x(t)$ might be greater than the amount that SRGENCO can really output, *i.e.*, $u(t) + \min\{z(t), \rho_d\}$.

2.3 Profit Model

By committing $x(t)$, the profit obtained by SRGENCO is $p(t)x(t)$. The consequence of over-commitment is captured in profit model by augmenting a penalty term. We adopt the penalty model in [20], in which the unit penalty payment in over-commitment is linearly proportional to the spot price $p(t)$ in the form of $\alpha_1 p(t) + \alpha_2$, where $\alpha_1, \alpha_2 \geq 0$ are constants.

Concluding above, the net profit obtained by SRGENCO at slot t, denoted by $R(t)$, is expressed as total profit subtracted by the (potential) penalty of the over-commitment, *i.e.*,

$$R(t) = p(t)x(t) - (\alpha_1 p(t) + \alpha_2)y(t). \tag{6}$$

2.4 Profit Maximization Problem

The objective is to maximize the cumulative profit obtained by SRGENCO over time horizon $\mathcal{T}$. The profit maximization offering strategy problem (HA-CSP) is formally casted as

$$\mathsf{HA} - \mathsf{CSP} \quad \max \quad \sum_{t \in \mathcal{T}} R(t), \quad \text{s.t. Eq. (2)},$$
$$\text{vars.} \quad \hat{p}(t) \in [p_{\min}, p_{\max}], \hat{x}(t) \geq 0, t \in \mathcal{T}.$$

In offline scenario, in which the values of $u(t)$ and $p(t)$ as the time-varying inputs to the problem are known ahead of time, the problem is a linear one which is easy to solve. We refer to [13, 18] as the related works that study related problems in offline settings. We note that in the offline scenario, the clearing prices are known to SRGENCO, hence it is not require to submit the offering price anymore. Consequently, the offering strategy reduces to announcing the commitment volume directly. In this way, the problem could be reformulated in an equivalent form with simpler structure.

In real-world, however, neither the renewable output $u(t)$ nor the clearing price $p(t)$ are revealed to SRGENCO when submitting the offer. Hence the focus in this paper is to tackle the problem in online setting, so, we formulate the problem in a way that SRGENCO submits both offering price and offering volume. Solving HA-CSP in online setting is challenging, since the problem is coupled over the time in the presence of the storage system. Recall that an important advantage of incorporating storage towards profit maximization is to (fully or partially) store the renewable supply in the storage when that market price is low, and discharge it when the market price is high. Without knowing the future values of $p(t)$ and $u(t)$, finding a profit maximization offering strategy, that implicitly determines how renewable supply and the stored electricity in the storage must be consumed is challenging.

Finally, we note that in [19, 20], following Markov decision process and approximate dynamic programming, different offering strategies are proposed given a particular probabilistic model of the clearing price and the renewable output. In these approaches, the solution is obtained in the sense of probabilistic expectation. In practice, however, real values might deviate from the underlying probabilistic models. Our general approach as explained in Sec. 2.5 has no assumptions on the stochastic modeling of the unknown time-varying inputs.

2.5 Our Approach: Online Competitive Algorithm Design

Our approach in this study is to follow competitive online algorithm design and propose online offering strategies in which the decision is made based on *only the current information*, and without any assumptions on the stochastic model on the renewable output and the clearing price. For the performance analysis, we use *competitive ratio* [12] which is a well-established metric to evaluate how good is the online solution.

Definition 1 *When the underlying problem is a profit maximization one, for an online algorithm* A*, its competitive ratio (*CR*) is defined as the maximum ratio between offline optimum and the profit obtained by* A*, over all inputs, i.e.,*

$$\mathrm{CR}(\mathsf{A}) \triangleq \max_{\boldsymbol{\omega} \in \Omega} \frac{R_{\mathsf{OFA}}(\boldsymbol{\omega})}{R_{\mathsf{A}}(\boldsymbol{\omega})}, \tag{7}$$

where $\boldsymbol{\omega} \in \Omega$ *refers to an instance of the online input parameters as*

$$\boldsymbol{\omega} \triangleq \big[\omega(t) = (u(t), p(t))\big]_{t \in \mathcal{T}}, \tag{8}$$

and Ω *is the set of all input instances. Moreover,* $R_{\mathsf{OFA}}(\boldsymbol{\omega})$ *and* $R_{\mathsf{A}}(\boldsymbol{\omega})$ *are the profits earned by the optimal offline solution and the online algorithm* A *respectively, when the input is* $\boldsymbol{\omega}$.

By this definition, the smaller the competitive ratio, the better the performance is, since it guarantees no matter what the input is, the online optimal strategy obtains the profit close to the offline optimum.

3 Optimal Online Offering Strategy with Accurate Single-Slot Prediction

In this section, we propose online competitive algorithms for a simplified version of HA-CSP, in which the accurate data for the next slot is available for both the renewable supply and the clearing price. Later in Sec. 4, based on the insights from result of this section on the simplified scenario, we tackle the general case and propose online algorithms with neither the renewable supply nor the clearing price known to srGENCO when submitting the bids.

3.1 Simplified Problem with Accurate Single-Slot Prediction

For the sake of simplification in design, we first assume that both $p(t)$ and $u(t)$ values for the next slot are revealed to srGENCO, perhaps by accurate short-term forecasting tools. In this way, HA-CSP is largely simplified in two ways:

(i) since $p(t)$ is known, the offering strategy reduces to finding just the commitment amount $x(t)$. We relax this assumption in Sec.4.1.

(ii) since $u(t)$ is known, all the inputs and variables in Eq. (5) are known to srGENCO when submitting the offer, hence the over-commitment never happens, thereby the penalty term in the objective of HA-CSP vanishes. We relax this assumption in Sec. 4.2.

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Since in the new setting the price $p(t)$ is known for the next slot, and to be consistent with the general formulation, we set $\hat{p}(t) = p(t)$ and following Eq. (1), we get $x(t) = \hat{x}(t)$. Then, given the above assumptions, the only optimization variable would be the commitment amount $x(t)$. Now, we cast the simplified offering strategy problem sOSP as

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\begin{aligned}
\textsf{sOSP} \quad \max \quad & \sum_{t \in \mathcal{T}} p(t) x(t) \\
\text{s.t.} \quad & x(t) \le \min\{z(t), \rho_D\} + u(t), \\
& z(t+1) = \Big[z(t) + x_c(t) - x_d(t)\Big]_{\mathcal{C}}, \\
\text{var}: \quad & x(t) \ge 0, t \in \mathcal{T},
\end{aligned}
$$

where the first constraint ensures that over-commitment never happens. And the second constrain involves the evolution of the storage level, where $x_c(t)$ and $x_d(t)$ defined in Eqs. (3)-(4) represent the charging and discharging amounts at time t.

Remarks. (1) Even though sOSP is simplified, it is still challenging and non-trivial in online setting. That is, for sOSP the forthcoming values for two inputs $p(\tau)$ and $u(\tau)$ for $\tau \in \{t+1, \dots, T\}$ are unknown, thereby the problem is still online, and again it cannot be decomposed across the time because of the evolution of the storage.

(2) sOSP is closely related to the classical *time series search* and *one-way trading* problems [16, 22].[3] As compared to these classical problems, sOSP can be characterized as a generalized version in two aspects: (i) sOSP introduces another exogenous parameter $u(t)$ in addition to the clearing price $p(t)$, this means that the adversary has more freedom to choose the worst-case input, this makes the competitive analysis more challenging; (ii) sOSP introduces the capacity constraint associated with the storage system. Putting together both differences, our investigations (see Sec. 3.3.1) demonstrate that neither *fixed-threshold* policy, *i.e.*, the policy that finds a fixed exchange rate and will accept the rates above that, nor the competitive analysis approach proposed in [16] yields the optimal solution for sOSP in this paper. Instead, the optimal strategy for sOSP admits an *adaptive-threshold-based* policy that achieves the optimal competitive ratio as a logarithmic function of θ, whose details are stated in the next section.

3.2 Online Algorithm Design for sOSP

In this section, we propose a simple online offering strategy (sOffer) to solve sOSP. Then we analyze its competitiveness and show that sOffer achieves the best competitive ratio.

3.2.1 High Level Intuitions

Intuitively, a proper offering strategy must consider two issues in decision making:

(i) the clearing price $p(t)$ for the incoming slot t, the higher the price, the more the SRGENCO is willing to commit,

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[3]In one-way trading problem, a trader needs to exchange from one currency to another currency, given a time-varying exchange rates arriving online. The trader can decide to accept the current price or wait for the more attractive prices in future. The time series search problem is also quite similar [16, 22].

Algorithm 1 —sOffer $\big[p(t), u(t), z(t)\big]$

1: $z^+(t) \leftarrow \min\{z(t) + u(t), C\}$
2: $\hat{p}(t) \leftarrow g(z^+(t))$; see Eqs. (9) and (10) for the optimal design of $g(z)$
3: calculate $\hat{x}(t)$ according to $\hat{p}(t)$, see Eq. (13) and the following analysis for the optimal design
4: $\hat{p}(t) \leftarrow p(t)$
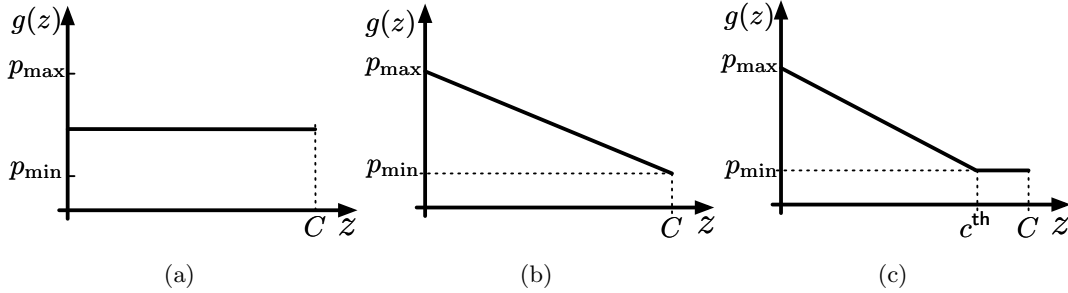5: submit offer $\big(\hat{p}(t), \hat{x}(t)\big)$;

Figure 2: Different structures of function $g(z)$.

(ii) the storage level $z(t)$, if the storage level is almost full, SRGENCO would be more interested to commit to have more capacity for the forthcoming slots to store the electricity. On the other hand, if the storage level is almost unoccupied, SRGENCO might keep this electricity to commit with higher price in future slots.

Putting together both clearing price and the storage level, we design our algorithm following an *adaptive threshold-based* strategy, since it adaptively changes the offering volume based on the current storage level and the clearing price.

3.2.2 Main Algorithm

The main idea is to construct a function $g(z) : [0, C] \rightarrow [p_{\min}, p_{\max}]$.[4] The input to function $g(\cdot)$ is the aggregation of the incoming renewable supply $u(t)$ and the current storage level $z(t)$, projected into the capacity of the storage, *i.e.*, $\min\{z(t) + u(t), C\}$. The output of $g(\cdot)$ is the *candidate offering price* $\hat{p}(t)$ for SRGENCO at slot t. Note that since we assume that SRGENCO knows clearing price $p(t)$, it calculates its candidate offering price $\hat{p}(t)$ and then based on the comparison between these two values decides how to submit its offer.

The function $g(\cdot)$ should be decreasing, *i.e.*, with the increase in the current storage level, the offering price would be decreased (see intuition (ii) above). Given function $g(z)$, designing sOffer is rather straightforward and is summarized as Algorithm 1. At slot t, first, sOffer calculates candidate offering price $\hat{p}(t)$ based on the given $g(z)$. Then, it calculates the offering volume, whose details is explained in Sec. 3.3.3. In Line 4, the offering price is set to $\hat{p}(t) = p(t)$ to ensure that $\hat{x}(t)$ is committed to market anyway. In next, we explain how to design function $g(z)$.

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[4]To be consistent to the notations in this paper, ideally we must denote the function as $g(z(t))$. However, for simplicity we drop the slot index t for $z(t)$.

3.3 The Design of Function $g(z)$

3.3.1 On the Importance of Designing Function $g(z)$

First, we highlight that $g(z)$ plays a critical role in sOffer, and the competitive ratio can be improved by optimizing function $g(z)$. To illustrate the impact of $g(z)$, we investigate the behavior of sOffer under different structures of function $g(z)$.

(i) $g(z) = c$, where $c \in [p_{\min}, p_{\max}]$, is a constant value as shown in Fig. 2(a). If $c > p_{\min}$, for the extreme case where $p(t) < c, t \in \mathcal{T}$, it could be easily shown that the profit obtained by the sOffer is always 0, thereby the competitive ratio would be ∞. On the other hand, $c = p_{\min}$, the competitive ratio is upper bounded by $p_{\max}/p_{\min}$, which is not intriguing, since sRGENCO always commits the electricity to the market regardless of the clearing price and, it loses the opportunity of utilizing the potentials of storage in obtaining more profit.

Consequently, an intelligent function design aims to improve the competitiveness by reserving the storage for forthcoming slots with higher clearing price. This goal can be achieved by adopting a decreasing function.

(ii) $g(z) = az + p_{\max}$, where $a < 0$, as depicted in Fig. 2(b). Again, under the extreme case, $\mathrm{CR}(\textsf{sOffer}) \to \infty$ if $p(t) = p_{\min}$. Note that this linear structure for $g(z)$ is one example and any strictly decreasing function with $g(0) = p_{\max}$ and $g(C) = p_{\min}$ behaves similarly in the worst case.

(iii) Another smart alternative is a piece-wise function as depicted in Fig. 2(c). Function $g(z)$ again is decreasing initially. However, after the storage level reaches to a threshold value c^{th}, $g(z)$ changes to a constant function, *i.e.*,

$$g(z) = \begin{cases} \hat{g}(z) & \text{if } z \leq c^{\text{th}}, \\ p_{\min} & z \geq c^{\text{th}}. \end{cases} \tag{9}$$

Now, finding the optimal function $g(z)$ reduces to finding $\hat{g}(z)$ and c^{th}. In the next, we introduce this function. Moreover in Sec. 3.4, we analyze the competitiveness of the algorithm and prove that the proposed function achieves the optimal CR.

3.3.2 Optimal Design of Function $g(z)$

The following theorem summarizes our main contribution in this section.

Theorem 3.1 *By setting $\hat{g}(z)$ in Eq. (9) as*

$$\hat{g}(z) = p_{\min} e^{\frac{(c^{\text{th}} - z)c^{\text{th}}}{C(C - c^{\text{th}})}}, \tag{10}$$

and c^{th} as

$$c^{\text{th}} = C - \frac{(2 + \log \theta)C - \sqrt{\log^2 \theta + 4 \log \theta C}}{2} > 0, \tag{11}$$

sOffer *achieves the optimal competitive ratio for* sOSP *as*

$$\mathrm{CR}(\textsf{sOffer}) = \frac{(2 + \log \theta) + \sqrt{\log^2 \theta + 4 \log \theta}}{2}. \tag{12}$$
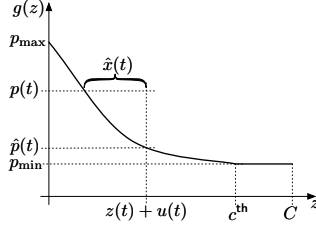
Figure 3: Illustration of calculating the offering volume $\hat{x}(t)$ when $p(t) > \hat{p}(t)$.

The proof is given as the competitive analysis in Sec. 3.4.

Remark. The theorem shows that the competitive ratio is proportional to a logarithmic function of θ as the ratio between the maximum and minimum clearing prices. In practice, the scale of θ varies from 2 to 50 in different markets, *e.g.*, the clearing prices in PJM [8] and NYISO [5] in July, 2015 is in [\$13.9, \$186.9] and [\$8.1, \$43.1] per MWh. Given $\theta = 50$, the competitive ratio is around 5.74. Our experimental results depict much lower empirical ratios using the real prices in different markets. For details we refer to Table 3.

3.3.3 Calculating the Offering Volume

Given the optimal function $g(z)$ in Theorem 3.1, we can finally find the offering volume $\hat{x}(t)$ as follows.

$$\hat{x}(t) = \begin{cases} [\rho_c - u(t)]^+, & \text{if } \hat{p}(t) > p(t), \\ z(t) + u(t) - \min\{c^{\mathsf{th}}, z(t) + \rho_c\}, & \text{if } \hat{p}(t) = p(t) = p_{\min}, \\ z(t) + u(t) - \min\{\hat{g}^{-1}(p(t)), z(t) + \rho_c\}, & \text{if } \hat{p}(t) < p(t), \end{cases} \tag{13}$$

where $\hat{g}^{-1}(p(t))$ is well-defined since $\hat{g}(z)$ defined in Eq. (9) is monotonically decreasing. More specifically, when $\hat{p}(t) > p(t)$, SRGENCO stores the renewable supply as much as it can, considering the charging rate constraint ρ_c, then, the residual is committed to the market. If $\hat{p}(t) = p(t) = p_{\min}$, it means that the storage level exceeds the threshold level c^{th}, so, SRGENCO offers the minimum price $p_{\min}$. Since the market price is also $p_{\min}$ it commits the electricity until the storage level reaches the threshold c^{th} or $z(t) + \rho_c$, *i.e.*, $\hat{x}(t) = z(t) + u(t) - \min\{c^{\mathsf{th}}, z(t) + \rho_c\}$. Finally, the last situation is $\hat{p}(t) < p(t)$, and $p(t) > p_{\min}$. In this case, we are in the exponential part of function $g(z)$ and $\hat{x}(t)$ is calculated as the total supply $u(t) + z(t)$ subtracted by $\min\{\hat{g}^{-1}(p(t)), z(t) + \rho_c\}$. An illustration of the offering volume in this case when $\hat{g}^{-1}(p(t)) \leq z(t) + \rho_c$ is depicted in Fig. 3. Finally, to capture the maximum discharge rate of the storage, it suffices to modify offering volume as $\hat{x}(t) = \min\{u(t) + \rho_d, \hat{x}(t)\}$.

3.4 Competitive Analysis

Our goal in this section is to design function $g(z)$ (especially $\hat{g}(z)$ and c^{th} in Eq. (9)) so as to minimize the competitive ratio. By doing so we prove the result in Theorem 3.1. To simplify our explanation in this section and without loss of generality, we quantize the energy-related variables $x(t)$ and $z(t)$ and the input parameters $u(t)$ and C to take the integer values. More specifically,

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considering that C is a real number, we can define discretized capacity C_d as

$$C_\mathsf{d} = C/\eta, \tag{14}$$

where η is the unit of electricity. In this way, C_d belongs to integer numbers. To avoid notation complexity, however, in our analysis in this section, we abuse notation C to denote the discretized C_d. Similarly, we abuse $x(t), z(t)$, and $u(t)$ as their discretized versions. Finally, we assume that initially the storage is full.

An immediate consequence of the above discretization is that function $g(z)$ could be considered as a step function. We assume that there are n steps for $g(z)$, each of which indexed by i, and $i \in \mathcal{N} = \{1, 2, \ldots, n\}$. Generally, the length of each step could be different, hence, let us denote by l_i as the length of step i. We can characterize the threshold value c^th defined in Eq. (9) as the storage capacity subtracted by the length of the last step, *i.e.*,

$$c^\mathsf{th} = C - l_n. \tag{15}$$

Moreover, we define $b_i = \sum_{k=1}^{i} l_k$ as the cumulative length until step i. Since $z \in [0, C]$ we get $C = b_n = \sum_{k=1}^{n} l_k$. Finally, we denote $p_i = g(b_i)$ as the threshold price when the storage level is b_i under function $g(\cdot)$. By this definition, we get $p_1 = p_{\max}$ and $p_n = p_{\min}$.

By $z_{\boldsymbol{\omega}}^{\mathsf{sOffer}}(t)$, we denote the storage level at slot t under sOffer, and a particular instance $\boldsymbol{\omega}$ as defined in Eq. (8). Let us denote the minimum storage level that sOffer reaches under $\boldsymbol{\omega}$ as $b \in \{0, 1, \ldots, C\}$, *i.e.*, $\min_{t \in \mathcal{T}} z_{\boldsymbol{\omega}}^{\mathsf{sOffer}}(t) = b$. Then, using this definition, we can partition the universal set of input instances Ω to multiple subsets as follows

$$\Omega = \bigcup_{b \in \{0, 1, \ldots, C\}} \Omega_b^{\mathsf{sOffer}},$$

where $\Omega_b^{\mathsf{sOffer}} \triangleq \left\{ \boldsymbol{\omega} \in \Omega : \min_{t \in \mathcal{T}} z_{\boldsymbol{\omega}}^{\mathsf{sOffer}}(t) = b \right\}.$

In particular, subset $\Omega_b^{\mathsf{sOffer}}$ represents the coalition of all input instances that results in the minimum storage level b upon executing the deterministic online algorithm sOffer.

Definition 2 *Define the local competitive ratio* $\mathrm{CR}_b(\mathsf{sOffer})$ *of* sOffer *under the subset of input instances* $\Omega_b^{\mathsf{sOffer}}$ *as*

$$\mathrm{CR}_b(\mathsf{sOffer}) \triangleq \max_{\boldsymbol{\omega} \in \Omega_b^{\mathsf{sOffer}}} \frac{R_{\mathsf{OFA}}(\boldsymbol{\omega})}{R_{\mathsf{sOffer}}(\boldsymbol{\omega})}. \tag{16}$$

Given Definition 2, we can redefine $\mathrm{CR}(\mathsf{sOffer})$ as follows.

Definition 3 *Define* CR *(*sOffer*) as the maximum of* $\mathrm{CR}_b(\mathsf{sOffer})$ *over all subsets* $\Omega_b^{\mathsf{sOffer}}, b \in \{0, 1, \ldots, C\}$, *i.e.*,

$$\mathrm{CR}(\mathsf{sOffer}) = \max_{b \in \{0, 1, \ldots, C\}} \mathrm{CR}_b(\mathsf{sOffer}). \tag{17}$$

The following lemma characterizes a closed-form of $\mathrm{CR}_b(\mathsf{sOffer})$ for $b \in \{b_1, b_2, \ldots, b_{n-1}\}$.

Lemma 3.2 *For* sOffer, *if* $\mathrm{CR}_{b_i}(\mathsf{sOffer}) \geq \mathrm{CR}_{b_k}(\mathsf{sOffer}), k \in \{i+1, i+2, \ldots, n-1\}$ *and* $i \leq n-1$, *then we have:*

$$\mathrm{CR}_{b_i}(\mathsf{sOffer}) = \frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k}{\sum_{k=i+1}^{n} p_k l_k}. \tag{18}$$

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otherwise,

$$\mathrm{CR}_{b_i}(\mathsf{sOffer}) \geq \frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k}{\sum_{k=i+1}^{n} p_k l_k}. \tag{19}$$

Through the proof of 3.2, we can justify that the competitive ratio takes maximum value only among the subsets with $b = b_i$, $i \in \{1, 2, \ldots, n-1\}$, *i.e.*,

$$\mathrm{CR}(\mathsf{sOffer}) = \max_{b \in \{b_1, b_2, \ldots, b_{n-1}\}} \mathrm{CR}_b(\mathsf{sOffer}).$$

Using the result in Lemma 3.2, we get the global competitive ratio of sOffer under universal set of instances as follows

$$\mathrm{CR}(\mathsf{sOffer}) = \max_{i \in \{1, 2, \ldots, n-1\}} \frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k}{\sum_{k=i+1}^{n} p_k l_k}. \tag{20}$$

Our goal is to achieve the minimum possible value for $\mathrm{CR}(\mathsf{sOffer})$. Our design space toward this goal is to find: (i) the optimal value for p_i which directly characterizes function $g(z)$, recall that by definition $p_i = g(b_i)$, and (ii) l_n as the length of the last step in function which characterizes the threshold level c^{th}, recall that we have $c^{\mathsf{th}} = C - l_n$.

The following lemma states that the minimum global competitive ratio is achieved when the value of local competitive ratios are all equal.

Lemma 3.3 *Given a fixed $l_1, l_2, \ldots, l_n$, $\mathrm{CR}(\mathsf{sOffer})$ minimizes only if the following expression holds:*

$$\frac{p_{n-1} C}{p_n l_n} = \frac{p_{n-2} C + p_{n-1} l_{n-1}}{p_n l_n + p_{n-1} l_{n-1}} = \cdots = \frac{p_1 C + \sum_{k=2}^{n-1} p_k l_k}{\sum_{k=2}^{n} p_k l_k}.$$

Using the result in Lemma 3.3 and by straightforward calculations, we can express l_i as

$$l_i = \frac{p_{i-1} - p_i}{p_i} \frac{p_n C l_n}{p_{n-1} C - p_n l_n}, i \in \{2, 3, \ldots, n-1\}. \tag{21}$$

Given $C = \sum_{i=1}^{n} l_i$, and combining with (21), we get

$$C = l_1 + \frac{p_n C l_n}{p_{n-1} C - p_n l_n} \sum_{i=2}^{n-1} \frac{p_{i-1} - p_i}{p_i} + l_n.$$

Lemma 3.4 *When $C \to \infty$, the competitive ratio of sOffer takes the minimum value only if $p_{n-1} \to p_n$.*

Recall that C is the discretized version of the original storage capacity, and $C \to \infty$ could be achieved if we choose sufficiently small unit of electricity η as in Eq. (14). Given the results in Lemma 3.3 and Lemma 3.4, we have

$$\begin{aligned} \mathrm{CR}(\mathsf{sOffer}) &= \frac{C}{l_n} = \frac{l_1}{l_n} + \frac{C}{C - l_n} \sum_{i=2}^{n-1} \frac{p_{i-1} - p_i}{p_i} + 1 \\ &\geq \frac{C}{C - l_n} \log \theta + 1. \end{aligned} \tag{22}$$

It can be verified that the above equation achieves the minimum value when $n \to \infty$, $l_1 = 0$, and $l_i = 1, i \in \{2, \ldots, n-1\}$. Now, if $z < C - l_n$, we have

$$C - z = \sum_{k=z}^{n} l_k = \frac{p_n C l_n}{p_{n-1} C - p_n l_n} \sum_{k=z}^{n-1} \frac{p_{k-1} - p_k}{p_k} + l_n$$
$$\approx \frac{C l_n}{C - l_n} \int_{p_{\min}}^{p_{z-1}} \frac{1}{p} dp + l_n$$
$$= \frac{C l_n}{C - l_n} \log \frac{p_{z-1}}{p_{\min}} + l_n,$$

where the second-to-the-last equality holds since the difference between p_i and p_{i-1} would be arbitrarily small when $n \to \infty$ and $l_i = 1, i \in \{2, \ldots, n-1\}$. Note that when $1 < z < C - l_n$ and $g(z) = p_{z-1}$, solving the above equation we get the following closed-form for $g(z)$

$$g(z) = \begin{cases} p_{\min} & \text{if } z \geq C - l_n, \\ p_{\min} e^{\frac{(C - l_n - z)(C - l_n)}{C l_n}} & \text{otherwise.} \end{cases} \tag{23}$$

Moreover, according to Eq. (22), we have

$$l_n = \frac{C}{\text{CR}(\mathsf{sOffer})} = \frac{C}{\frac{C}{C - l_n} \log \theta + 1},$$

and the closed form for l_n is

$$l_n = \frac{(2 + \log \theta)C - \sqrt{\log^2 \theta + 4 \log \theta} C}{2}. \tag{24}$$

Putting together the results in Eqs. (23), (24), and (15), the result in Theorem 3.1 is proved.

4 Online Offering Strategy without Accurate Single-Slot Prediction

In Sec. 4.1, we first extend the previous result to the case that the clearing price $p(t)$ is unknown to SRGENCO when submitting the offer, however, the renewable output $u(t)$ is known accurately. Second, in Sec. 4.2, we extend the result to the general case that the renewable output is known to SRGENCO with forecasting error.

4.1 mOffer: sOffer with multiple offer submissions; $p(t)$ Is Unknown, $u(t)$ Is Known

Our general approach in this scenario is to use the potentials of submitting *multiple offers* which is allowed in the current markets, *e.g.*, at most 10 offers are permitted for SRGENCO in PJM market [4]. Our approach in this case is to calculate total feasible commitment volume, and then partition this total amount into multiple offers, each of which conveying a portion of the offering volume, in different prices. In Algorithm 2, we summarize the detail of mOffer, as the multiple version of sOffer. In mOffer, we denote $\mathcal{B} = \{(p_1, x_1), (p_2, x_2), \ldots, (p_m, x_m)\}$ as the set of multiple offers submitted by the SRGENCO, where m is the maximum number of offers that is permitted by the market operator. In mOffer, if $\min\{u(t), \rho_c\} + z(t) > c^{\text{th}}$, it means that the current storage

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Algorithm 2 —mOffer $\big[u(t), z(t)\big]$

1: declare $\mathcal{B} \leftarrow \emptyset$ as the set of offers
2: **if** $\min\{u(t), \rho_c\} + z(t) > c^{\mathsf{th}}$ **then**
3:    $\mathcal{B} \leftarrow \big\{(u(t)+z(t)-c^{\mathsf{th}}, p_{\min})\big\}$
4:    $\Delta x \leftarrow \min\{c^{\mathsf{th}}, u(t) + \rho_d\}/(m-1)$
5:    **for** $i \leftarrow 1$ to $m-1$ **do**
6:        $p_i \leftarrow g(c^{\mathsf{th}} - i\Delta x)$
7:        $\mathcal{B} \leftarrow \mathcal{B} \cup \{(\Delta x, p_i)\}$
8:    **end for**
9: **else**
10:    $\mathcal{B} \leftarrow \{([u(t) - \rho_c]^+, p_{\min})\}$
11:    $\Delta x \leftarrow [u(t) + \min\{z(t), \rho_d\}]/(m-1)$
12:    **for** $i \leftarrow 1$ to $m-1$ **do**
13:        $p_i \leftarrow g(z(t) + u(t) - i\Delta x)$
14:        $\mathcal{B} \leftarrow \mathcal{B} \cup \{(\Delta x, p_i)\}$
15:    **end for**
16: **end if**
17: submit $\mathcal{B}$

---

level $z(t)$ plus the chargeable renewable supply, *i.e.*, $\min\{u(t), \rho_c\}$, exceeds the threshold value c^{th}, so, we can safely submit the surplus amount with the minimum price (Line 3), and partition the remaining amount equally in $m-1$ offers (Lines 4-8). Otherwise, we submit the excessive energy $[u(t) - \rho_c]^+$ at price $p_{\min}$ and partition the residual equally into $m-1$ offers (Lines 10-15). Theorem 4.1 characterizes the competitive ratio of mOffer as a function of CR (sOffer) and the number of submitted offers m.

Theorem 4.1 *The competitive ratio of* mOffer *is bounded by*

$$\mathrm{CR}(\mathsf{mOffer}) \leq \left(1 + \frac{\mathrm{CR}(\mathsf{sOffer})\theta}{m^2}\right) \mathrm{CR}(\mathsf{sOffer}).$$

Note that $\mathrm{CR}(\mathsf{mOffer}) \to \mathrm{CR}(\mathsf{sOffer})$ as $m \to \infty$. In experiments, we evaluate the impact of the number of offers on the performance of mOffer.

4.2   gOffer: generalized mOffer; $u(t)$ Is Given with Forecasting Error, $p(t)$ Is Unknown

Finally, we release the accurate forecasting assumption of the renewable output, and assume that the error of power generation in the forthcoming slot is bounded in a particular region. Namely the input to this algorithm is $\tilde{u}(t)$ as the predicted value and $e(t)$ as the maximum error, such that

$$u(t) \in [(1 - e(t))\tilde{u}(t), (1 + e(t))\tilde{u}(t)], \ 0 \leq e(t) \leq e_{\max}.$$

Our algorithm gOffer is a simple extension of mOffer, with replacing $u(t)$ with $(1 - e(t))\tilde{u}(t)$ as the minimum possible value for the renewable output. We skip the other details since they are the same as mOffer. Note that with this input, the algorithm behaves in the most conservative way, such that the over-commitment never happens. Extending the algorithm to the more aggressive
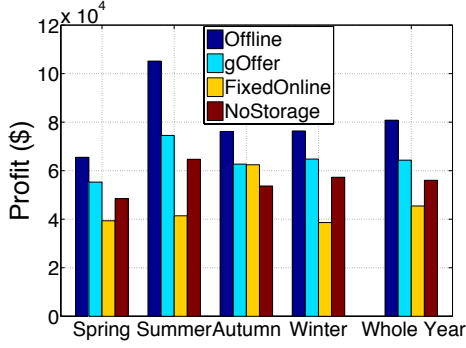
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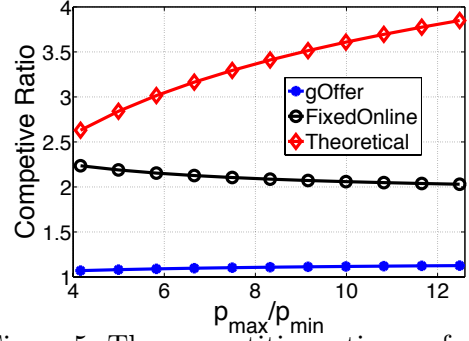Figure 4: The performance of different algorithms in different seasons

Figure 5: The competitive ratio as a function of price volatility

cases that takes into account the risk of over-commitment is part of our future work. In following theorem we characterizes the competitive ratio of gOffer.

Theorem 4.2 *Assuming $e_{\max} < 0.5$, the competitive ratio of gOffer is bounded by* $\mathrm{CR}(\mathsf{gOffer}) \leq \frac{1}{1-2e_{\max}}\mathrm{CR}(\mathsf{mOffer})$.

The proof for Theorem 4.2 is very simple. It's obvious that the amount of committed energy by gOffer is always larger than $(1 - 2e_{\max})x_{\mathsf{mOffer}}$ for any $t \in \mathcal{T}$.

Clearly, as $e_{\max} \to 0$, we get $\mathrm{CR}(\mathsf{gOffer}) \to \mathrm{CR}(\mathsf{mOffer})$. In experiments, we investigate the impact of forecasting error on the result of gOffer.

5 Experimental Results

In this section, we evaluate the performance of our online strategies using the real-word traces for the renewable output and electricity market prices. Our objective is two-fold: (i) to compare the performance against the optimal offline, a comparison algorithm [22], and a baseline in which there is no storage, and (ii) to investigate the impact of the system model and algorithm parameters.

5.1 Experimental Settings

5.1.1 Renewable Output and Electricity Market Prices

We use the wind generation data from PJM energy market [8] with the capacity of 10MW. We note that this is a wind farm in moderate size and the world largest has an operational capacity of 1020MW [11]. The hourly electricity price data are from PJM market for most of the experiments. We also demonstrated our results for Nord Pool [6] and NYISO [5] markets in Table 3. We note that the prices exhibit severe seasonal patterns. In particular, the prices are highly volatile, *e.g.*, in summer the peak price reaches as high as \$396.9/MWh, in 2015. For this reason we evaluate the performance of our algorithms in different seasons as well.

Table 2: Summary of comparison algorithms

Algorithm	Description
Our algorithms	
sOffer	Simplified online offering strategy; $p(t)$ and $u(t)$ are known
mOffer	sOffer with multiple submissions; $p(t)$ is unknown, $u(t)$ is known
gOffer	Generalized mOffer; $p(t)$ is unknown, $u(t)$ is known with error
Comparison Algorithms	
OFA	Optimal offline solution with storage
NoStorage	Optimal offline solution without storage
FixedOnline	Simple online algorithm with fixed threshold price [22]

5.1.2 Storage Capacity

Unless otherwise specified, the storage capacity is set to 20MWh. The maximum charge and discharge rates are 10MW. In reality, large scale Compressed Air Energy Storage (CAES) with similar parameters has been developed to cope with renewable uncertainty [23].

5.1.3 Parameters for the Algorithms

Unless otherwise specified, for mOffer and gOffer, the default value for the number of offers is 10. This is the common practice in PJM market [4]. Moreover, the maximum forecasting error $e_{\max}$ in gOffer is set to 10%. Finally, we note that each data point in figures is the average results of 100 different runs of the algorithms with $T = 360$ hours.

5.1.4 Comparison Algorithms

We compare the performance of our algorithm gOffer with three other alternatives: (1) OFA, the optimal offline solution that is implemented as the benchmark to obtain the empirical competitive ratio; (2) NoStorage, the optimal offline cost when there is no storage. This is used to evaluate the economic advantage of integrating the storage; (3) FixedOnline, another simple online algorithm with a fixed threshold price. Specifically, we follow the approach in [22] and set the threshold of this simple online algorithm fixed at $\sqrt{p_{\min}p_{\max}}$, regardless of the storage level. In this algorithm, srGENCO commits all the electricity whenever the price is not smaller than this threshold. The acronyms for all the algorithms are summarized in Table 2.

Table 3: Summary of Theoretical and Empirical Competitive Ratios on Different Electricity Markets

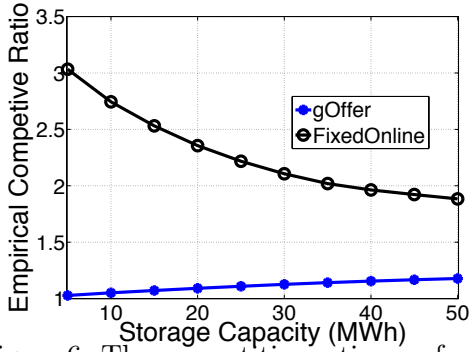| Electricity market	$\theta = p_{\max}/p_{\min}$	Theoretical CR	Empirical CR
PJM	13.44	4.37	1.18
NYISO	5.32	3.38	1.14
Nord Pool	3.63	2.95	1.09

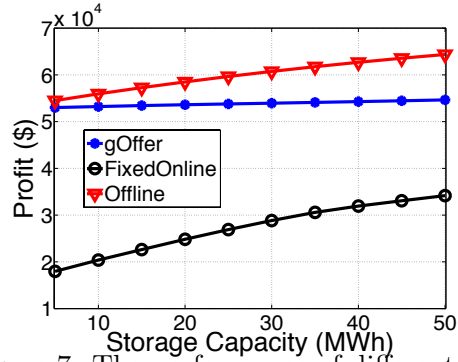Figure 6: The competitive ratio as a function of storage capacity

Figure 7: The performance of different algorithms as a function of capacity

5.2 Experimental Results

5.2.1 Comparison Results across Different Seasons

In this experiment we report the profit obtained by different algorithms in different seasons as well as the whole year. The result is depicted in Fig. 4. The main observations are: (1) gOffer achieves 80% of the offline optimum, which shows that it is close to optimal. (2) gOffer outperforms NoStorage by 15%, which signifies the substantial economic benefit of incorporating the storage. (3) gOffer outperforms FixedOnline by 42%, which depicts the superiority of our online algorithms as compared to other "storage-level-oblivious" online alternatives.

5.2.2 Impact of the Price Volatility

The electricity price in the deregulated electricity market exhibits large fluctuation. Theoretically, large price volatility will degrade the performance of the online algorithm, as the competitive ratio is an increasing function of $\theta = p_{\max}/p_{\min}$. In this experiment, we present the result under different values of θ. As shown in Fig. 5, gOffer is robust to price fluctuation with less than 5% increment, even though the theoretical competitive ratio increases by 44%. Meanwhile, we note that the empirical competitive ratio of FixedOnline decreases slightly as the θ increases. However, it is on average 90% larger than that of gOffer, which further signifies the superiority of gOffer. In addition, we report the result of gOffer for the prices in different markets in Table 3. The result signifies that the larger the price volatility, the large theoretical and empirical competitive ratios.
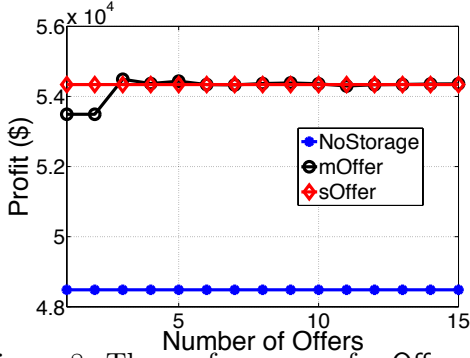
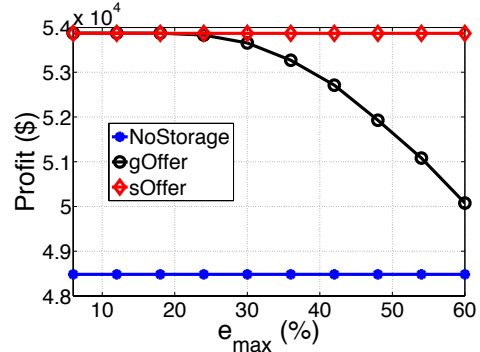Figure 8: The performance of mOffer as the number of bids increases

Figure 9: The performance of gOffer as forecasting error increases

5.2.3 Impact of the Storage Capacity

Storage capacity planning is an important issue that SRGENCO's owner needs to consider, since the storage is still expensive with the current technology. In this experiment, we vary the storage capacity from 5 to 50MWh to investigate its impact on the profit of SRGENCO. Fig. 6 and Fig. 7 show the empirical competitive ratios and the obtained profits, respectively. As the storage capacity increases, an increase in profit of both online algorithms is observed. However, the increase in gOffer is smaller (3%) than that of FixedOnline (90%). This is mainly because FixedOnline is completely oblivious to the storage level, and with the increase in capacity, there would be more room to mitigate this unawareness. Meanwhile, the empirical competitive ratio of gOffer increases with large storage capacity (from 1.03 to 1.18). This result depicts that when the capacity is in the order of the renewable capacity (say, ×0.5 to ×2), gOffer is close-to-optimal. However, when the storage capacity is much higher than the renewable capacity (say, ×5), perhaps more sophisticated algorithms are required.

5.2.4 Impact of the Uncertainty of the Clearing Price and the Renewable Output

In the last set of experiments we investigate the impact of the number of offers in mOffer (in Fig. 8) and the forecasting error in gOffer (in Fig. 9). In mOffer, we relaxed the assumption of sOffer and extend it to the case that the clearing price $p(t)$ is unknown. We proposed to submit multiple offers to alleviate its negative impact. To investigate how many offers are sufficient for mOffer to achieve the same performance level as sOffer, in Fig. 8, we vary the number of offers from 1 to 15. The notable observation is that submitting 1 or 2 offers is not sufficient. However, with 3 or more offers the performance is quite similar to sOffer in which the price is known in advance. In the last experiment, we increase the maximum error of renewable output e_{max} and calculate the profit of gOffer. The result shows that gOffer is robust to forecasting error that belows 20%, and the obtained profit decreases rapidly as error increases beyond 20%. Concluding above, these experiments demonstrate that the negative impact of the uncertainty in the clearing price can be effectively mitigated by multiple offer submissions. However, accurate short-term renewable forecasting is vital for SRGENCO to obtain a desired profit, since the errors higher than 20% can severely degrade the performance.

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6 Conclusion

This paper proposes competitive offering strategies for maximizing the profit of a renewable power producer with storage in hour-ahead market. The underlying problem is coupled over time and the input to the problem is revealed in an online fashion, both of which make the problem challenging. We first tackle a basic setting and propose algorithm for it. And in two successive steps we relax the basic assumptions and generalize our solutions and analysis to the realistic scenario. Our analytical results characterize competitive ratios for all the proposed algorithms. Finally, trace-driven evaluations demonstrate the close-to-optimal performance of our algorithms.

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.1 proof of Lemma 3.2

Lemma .1 *Under some worst case ω^*, assume $\min_{t \in \mathcal{T}} z_{\omega^*}^{\mathsf{sOffer}}(t) = b$ then we have $p(t) \le g(b) + \sigma$ for any $\sigma > 0$.*

Proof: Assume there's a time slot that $p(t) \ge g(b) + \sigma$ under some worst case ω^*. If $x_{\omega^*}^{\mathsf{OFA}}(t) \le x_{\omega^*}^{\mathsf{sOffer}}(t)$, we just construct another worst case which replaces the $t-$th input $(p(t), u(t))$ with the following input sequence:

$$p'(t) = g(z_{\omega^*}^{\mathsf{sOffer}}(t) + u(t)) + \sigma, \ u'(t) = u(t),$$
$$p'(t+1) = g(z_{\omega^*}^{\mathsf{sOffer}}(t+1)) + \sigma, \ u'(t+1) = 0,$$
$$p'(t+2) = g(z_{\omega^*}^{\mathsf{sOffer}}(t+2)) + \sigma, \ u'(t+2) = 0,$$
$$\vdots$$
$$p'(t+N) = g(z_{\omega^*}^{\mathsf{sOffer}}(t) + u(t) - x_{\omega^*}^{\mathsf{sOffer}}(t)), \ u'(t+N) = 0$$

where $N = \lceil x_{\omega^*}^{\mathsf{sOffer}}(t)/\sigma \rceil$. In this way, the profit earned by sOffer decreases while that of OFA keep unchanged. This contradicts the assumption at the beginning. Then we complete the proof. □

We adopt a worst-case analysis approach to prove Lemma 3.2. We assume the state of storage under online algorithm reaches the minimum value b_i at time T_1 and keeps unchanged until time T_2.

If there's no injection into the storage during $[1, T_2]$, according to Lemma .1, the maximum market clearing price $[1, T_1]$ is $g(b_i)$ and the maximum profit earned by OFA during $[1, T_1]$ is $g(b_i)C$. The

minimum profit earned by the online algorithm is $\sum_{k=b_i+1}^{b_n} g(k)$, so it's immediate that the maximum profit ratio is $g(b_i)C/\sum_{k=b_i+1}^{b_n} g(k)$.

(a) If there's energy stored into the battery during $[1, T_2]$, we can assume the last time interval with energy stored is $T_I = [T_3, T_4] \subset [1, T_2]$. During T_I, the state of storage under sOffer will keep increasing or unchanged. Then the off-line algorithm will commit $X = \sum_{t \in T_I} u(t)$ amount of energy before T_3 with lower price than $g(b_i)$. Moreover, we assume at time T_5, the state of storage under sOffer reaches $z_{\mathsf{sOffer}}^{\omega^*}(T_3)$ for the last time during $[1, T_2]$. Then the committed energy during $[T_3, T_5]$ by sOffer is equal to that by OFA.

If

$$\frac{g(z_{\mathsf{sOffer}}^{\omega^*}(T_3) - 1)X}{\sum\limits_{k=z_{\mathsf{sOffer}}^{\omega^*}(T_3)+1}^{z_{\mathsf{sOffer}}^{\omega^*}(T_3)+X} g(k)} > \mathrm{CR}_{b_i}(\mathsf{sOffer})$$

we can easily get that:

$$\mathrm{CR}_{\Omega_{z_{\mathsf{sOffer}}^{\omega^*}(T_3)}^{\mathsf{sOffer}}}(\mathsf{sOffer}) \geq \frac{(g(z_{\mathsf{sOffer}}^{\omega^*}(T_3)) + 1)z_{\mathsf{sOffer}}^{\omega^*}(T_3)}{\sum\limits_{k=b_n}^{x_{\mathsf{sOffer}}^{\omega^*}(T_3)} g(k)}$$

$$\geq \frac{(g(z_{\mathsf{sOffer}}^{\omega^*}(T_3)) + 1)X}{\sum\limits_{k=z_{\mathsf{sOffer}}^{\omega^*}(T_3)+1}^{z_{\mathsf{sOffer}}^{\omega^*}(T_3)+X} g(k)}$$

$$> \mathrm{CR}_{\Omega_{b_i}^{\mathsf{sOffer}}}(\mathsf{sOffer})$$

Thus, for the worst case, if the maximum profit ratio within the case set $\Omega_{b_i}^{\mathsf{sOffer}}$ satisfies $\mathrm{CR}_{\Omega_{b_i}^{\mathsf{sOffer}}}(\mathsf{sOffer}) \geq \mathrm{CR}_{\Omega_{b_k}^{\mathsf{sOffer}}}(\mathsf{sOffer})$ for $k = i+1, i+2, \ldots, n-1$, there's no stored energy during $[1, T_2]$ under the worst case.

(b) If there'is energy committed by sOffer during $[T_3 + 1, T]$ under the worst case, we can assume the period with energy committed during $[T_3 + 1, T]$ by sOffer is $T_D = [T_6, T_7]$. And assume the time that the state of battery first reaches $b_0 = z_{\mathsf{sOffer}}^*(T_7)$ at time T_8. It's easy to verify that, at time T_7 and T_8, the state of storage under OFA is 0. So the committed energy by OFA algorithm during $[T_3 + 1, T_7]$, denoted by X' is equal to that by sOffer. During T_D, the profit earned by online algorithm is at least $\sum_{k=z_{\mathsf{sOffer}}^{\omega^*}(T_6)-X'}^{z_{\mathsf{sOffer}}^{\omega^*}(T_6)} g(k)$, and the maximum profit earned by the OFA is $(g(z_{\mathsf{sOffer}}^{\omega^*}(T_6)) + X')X'$.

If

$$\frac{(g(z_{\mathsf{sOffer}}^{\omega^*}(T_7)) - 1)X'}{\sum\limits_{k=z_{\mathsf{sOffer}}^{\omega^*}(T_6)-X'}^{z_{\mathsf{sOffer}}^{\omega^*}(T_6)} g(k)} > \mathrm{CR}_{\Omega_{b_i}^{\mathsf{sOffer}}}(\mathsf{sOffer})$$

we can easily get that:

$$\mathrm{CR}_{\Omega^{\mathsf{sOffer}}_{z^{\omega^*}_{\mathsf{sOffer}}(T_6)}}(\mathsf{sOffer}) \geq \frac{(g(z^{\omega^*}_{\mathsf{sOffer}}(T_6))-1)z^{\omega^*}_{\mathsf{sOffer}}(T_6)}{\displaystyle\sum_{k=b_n}^{z^{\omega^*}_{\mathsf{sOffer}}(T_6)} g(k)}$$

$$\geq \frac{(g(z^{\omega^*}_{\mathsf{sOffer}}(T_6))-1)X'}{\displaystyle\sum_{k=z^{\omega^*}_{\mathsf{sOffer}}(T_6)-X'}^{z^{\omega^*}_{\mathsf{sOffer}}(T_6)} g(k)}$$

$$> \mathrm{CR}_{\Omega^{\mathsf{sOffer}}_{b_i}}(\mathsf{sOffer})$$

Thus, for the worst case, if the maximum profit ratio within the case set $\Omega^{\mathsf{sOffer}}_{b_i}$ satisfies $\mathrm{CR}_{\Omega^{\mathsf{sOffer}}_{b_i}}(\mathsf{sOffer}) \geq CR_{\Omega^{\mathsf{sOffer}}_{b_k}}(\mathsf{sOffer})$ for $k = i+1, i+2, \ldots, n-1$, there's no energy committed during $[T_3+1, T]$ by sOffer, and the maximum profit gained by the off-line algorithm is $\sum_{k=b_{i-1}+1}^{b_n-1} g(k)$.

Concluding (a) and (b), we complete the proof.

.2 proof of Lemma 3.3

Given $l_1, l_2, \ldots, l_n$, solving (20) can be formulated as the following problem:

$$
\begin{aligned}
\min : \quad & y \\
\text{subject to:} \quad & y \geq \frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k}{\sum_{k=i+1}^{n} p_k l_k} \quad i = 2, 3, \ldots, n-1 \\
& p_{\min} \leq p_i \leq p_{\max} \quad i = 2, 3, \ldots, n-1 \\
\text{var :} \quad & y, \ p_i \quad\quad\quad\quad\quad i = 2, 3, \ldots, n-1
\end{aligned}
$$

The hessian matrix of the non-equality constraints is:

$$
\begin{pmatrix}
0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & -l_{i+1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & -l_{n-1} \\
0 & \cdots & 0 & -l_{i+1} & \cdots & -l_{n-1} & 0
\end{pmatrix}
$$

Given $l_k > 0$ for $k = 1, 2, \cdots, n-1$, the hessian matrix is positive semi-definite. Then we can verify that the nonlinear constraints are convex.

By partially dualizing on the first set of non-equality constrains, we can get the following Lagrangian function:

$$L(y, \boldsymbol{\mu}) = y + \sum_{i=2}^{n-1} \mu_i (p_i C + \sum_{k=i+1}^{n-1} p_k l_k - y \sum_{k=i}^{n} p_k l_k)$$

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where μ_i, $i = 2, 3, \ldots, n-1$ are the dual variables associated with the non-equality constraints.

Then the first-order optimality necessary condition is:

$$1 - \sum_{i=2}^{n-1} \mu_i \sum_{k=2}^{n} p_k l_k = 0$$

$$\mu_{i+1} C - l_i \sum_{k=2}^{i} (y-1)\mu_k = 0, \ i = 2, 3, \ldots, n-1$$

Note that $y > 1$, $0 < l_i < C$ and $p_{\min} \leq p_i \leq p_{\max}$ for $i = 2, 3, \ldots, n-1$. Then by above equation, we can derive that $\mu_i > 0$, $i = 2, 3, \cdot, n-1$. Moreover, according to the complementary slackness condition:

$$\mu_i \left(p_{i-1} C + \sum_{k=i}^{n-1} p_k l_k - y \sum_{k=i}^{n} p_k l_k \right) = 0, \ i = 2, 3, \ldots, n-1$$

we can get that the followings always hold:

$$p_{i-1} C + \sum_{k=i}^{n-1} p_k l_k - y \sum_{k=i}^{n} p_k l_k = 0, \ i = 2, 3, \ldots, n-1$$

or

$$\frac{p_{i-1} C + \sum_{k=i}^{n-1} p_k l_k}{\sum_{k=i}^{n} p_k l_k} = y, \ i = 2, 3, \ldots, n-1$$

Then we complete the proof.

.3 proof of Lemma 3.4

Proof: If $p_{n-1} \nrightarrow p_n$ for the optimal $g(z)$, we can assume there's another function $g'(z)$ which contains $n + 1$ steps. We let $g'(z) = g(z)$ for $z = 1, 2, \ldots, b_{n-1}, b_{n-1} + 1, b_{n-1} + 2, \ldots, b_n$ and $g'(b_{n-1}+1) = (p_{n-1}+p_n)/2$. According to Lemma 3.2, easily we can get that the competitive ratio with $g'(z)$ under the subsets satisfy the followings:

$$\frac{\frac{p_{n-1}+p_n}{2} C}{p_1 (l_n - 1)} < \frac{p_2 C}{p_1 l_n}$$

$$\frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k + \frac{p_{n-1}+p_n}{2}}{\sum_{k=i+1}^{n-1} p_k l_k + \frac{p_{n-1}+p_n}{2} + p_1 (l_n - 1)} < \frac{p_i C + \sum_{k=i+1}^{n-1} p_k l_k}{\sum_{k=i+1}^{n} p_k l_k}$$

That means the competitive ratio with $g'(z)$ is smaller than that with $g(z)$, contradicting the assumption that $g(z)$ is the optimal. This completes the proof. $\qquad\square$

.4 proof of Theorem 4.1

Proof: Following similar lines with Lemma 3.3 and .1, we can find that the worst instance for mOffer is similar to sOffer. For any worst instance ω, if $z^\omega_{\text{mOffer}}(t) > z^\omega_{\text{sOffer}}(t)(1 + 1/m)$, it can

be verified that $x_{\mathsf{sOffer}}(t) < x_{\mathsf{mOffer}}(t)$, $t \in \mathcal{T}$ must hold according to the algorithm rules. So the cumulative profit difference between mOffer and sOffer at time slot t is bounded by $p(t)z^{\omega}_{\mathsf{sOffer}}(t)/m$ under ω. So we have:

$$\mathrm{CR}(\mathsf{mOffer}) \leq \max_{t \in \mathcal{T}} \frac{R_{\mathsf{OFA}}(\omega) + p(t)z^{\omega}_{\mathsf{sOffer}}(t)/m}{R_{\mathsf{sOffer}}(\omega)}$$

$$\leq \mathrm{CR}(\mathsf{sOffer}) + \max_{t \in \mathcal{T}} \frac{p(t)z^{\omega}_{\mathsf{sOffer}}(t)/m}{R_{\mathsf{sOffer}}(\omega)}$$

Moreover, for any time slot t, the following inequality always hold:

$$\frac{p(t)z^{\omega}_{\mathsf{sOffer}}(t)/m}{R_{\mathsf{sOffer}}(\omega)} \leq \frac{g((1-1/m)z^{\omega}_{\mathsf{sOffer}}(t))z^{\omega}_{\mathsf{sOffer}}(t)/m}{\int_{z^{\omega}_{\mathsf{sOffer}}(t)}^{C} g(z)dz}$$

$$\leq \frac{g((1-1/m)c^{\mathsf{th}})(C - c^{\mathsf{th}})/m}{\int_{c^{\mathsf{th}}}^{C} g(z)dz}$$

$$\leq \frac{\theta \mathrm{CR}(\mathsf{sOffer})}{m^2}$$

The above inequality use the result that $p(t) \leq g((1-1/m)z^{\omega}_{\mathsf{sOffer}}(t))$ under the worst case ω and that $g(z)$ is convex. $\qquad \square$