Optimal Online Algorithms for Peak-Demand Reduction Maximization with Energy Storage

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ABSTRACT

We consider an emerging scenario where large-load customers employ energy storage (e.g., fuel cells) to reduce the peak procurement from the grid, which accounts for up to 90% of their electricity bills. We focus on maximizing the peak-demand reduction, which directly captures the economic benefits of using energy storage for the purpose. While the problem is easy to solve under the (ideal) offline setting where the electricity demands are known beforehand, it turns into a challenging online decision-making problem under the more practical online setting, where the demands are revealed sequentially but one has to make irrevocable discharging decisions without knowing future demands. In this paper, we develop an optimal online algorithm for the problem that achieves the best possible *competitive ratio* (CR) among all (deterministic and randomized) online algorithms. We solve a linear number of linear-fractional problems to find the best CR in polynomial time. We then extend our algorithm to an adaptive one with improved average-case performance and the same optimal worst-case performance. Simulation results based on real-world traces show that, under typical settings, our algorithms achieve up to 81% peak reduction attained by the optimal offline solution and 20% more peak reduction than baseline alternatives.

CCS CONCEPTS

• **Computing methodologies → Planning under uncertainty**; • **Theory of computation → Online algorithms**; • **Applied computing → Decision analysis**.

KEYWORDS

energy storage management, peak-demand charge, online competitive algorithms

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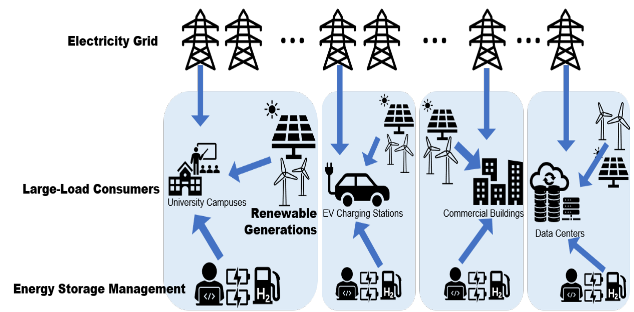


Figure 1: Illustrative scenarios where large-load consumers own renewable generations and energy storage systems.

1 INTRODUCTION

As a typical demand response program [11], utilities exploit delicate pricing schemes for motivating customers to modify their power consumption patterns. A large-load consumer's electricity bill usually consists of two parts: volume charge and demand charge [6]. The volume charge depends on the amount of consumed energy and time-based rates like time-of-use (TOU) pricing [27], reflecting the energy cost over time. These pricing strategies promote load shifting to the off-peak hours with cheaper unit prices, but may cause rebound peaks, highlighting the second part of the bill [8].

The peak-demand charge, as a punitive charge on the maximum power consumption during a billing period, motivates large-load customers to flatten their demand curves [6]. It is calculated by the average power over a specified interval, e.g., 15 or 30 minutes. According to [10], the demand charge rate and on-peak volume charge rate are around 118 HK dollars/kW and 55 HK cents/kWh respectively. It follows that the cost of lifting the peak demand is over 200 times that of increasing the off-peak energy consumption by one unit. Then, it is no surprise that the peak-demand often takes up a large portion of the bill, e.g., up to 90% for DC fast charging stations [18] and 80% for Google data centers [40]. Thus, for electricity cost saving, customers should pay particular attention to their peak demands.

Two approaches to electricity cost saving are directly shifting partial loads from an on-peak period to an off-peak one and reducing peak demands [21, 31, 40]. However, such methods are not always applicable, as certain loads cannot be shifted or cut. Meanwhile, bulk energy consumers increasingly invest in self-owned renewable generations, e.g., solar power systems, for green buildings and industries. These installations can reduce the amount of energy purchased from the grid. However, they may result in a more fluctuating net demand curve and not reduce the peak-demand charge because the renewable generations are highly volatile [15].

On the other hand, customers can utilize energy storage to reshape their electricity procurements from the grid, without directly changing their consumption profiles. The rapid development of storage technologies makes it convenient and economical to build and maintain a storage system. Energy storage has been useful to meet demand surges and provide uninterrupted power supply in power systems [12, 32, 35]. It will also play a part in reducing the electric bill, especially the peak-demand charge of a large-load user.

Motivated by these observations, we are interested in maximizing the peak-demand reduction by discharging energy storage in an on-peak period, for a large-load customer with self-owned renewable generation. See Fig. 1 for illustrative scenarios. Note that we consider a general storage system where recharging by the grid or the renewable generation may not be available, like fuel cells.

Knowing the future demands, we can easily make the best of energy storage. Unfortunately, the small-scale demands are highly uncertain, and self-owned renewable generations even exacerbate the volatility of demands over time [15]. Thus, we have to deal with the online setting, where an input sequence will be revealed sequentially in time, and the decision maker has to causally make irrevocable decisions at current time with less or no future information. In our problem, an input sequence refers to a series of demands over time and the operator causally determines the usage of the limited energy storage in each time interval. Furthermore, we are challenged by the non-cumulative nature of peak consumption and the coupling of online decisions to maximize the peak-demand reduction by using the energy storage in real-time.

In our scenario, stochastic optimization or model predictive control may be vulnerable to the inaccuracy of estimated models and sensitive to a volatile practical environment [1, 22, 28, 29, 35]. Although robust optimization relies little on reliable predictions, it seldom deals with sequential decision making. Its focus on the worst-case outcome over an uncertainty set often makes the resulting algorithms computationally expensive and conservative [3, 39].

In our paper, we apply another popular and useful tool – online algorithm design with competitive analysis [4, 7]. While stochastic optimization and robust optimization emphasize the absolute performance, competitive online optimization concerns the relative performance compared to the optimal offline outcomes derived with perfect information. Specifically, it evaluates an online algorithm via competitive ratio (CR), relating to the worst-case ratio between the optimal offline and online outcomes under the same input sequence. Then, the best possible CR among all online algorithms captures the price of uncertainty for the considered online problem. Like robust optimization, competitive online optimization requires little accurate predictions. Moreover, its worst-case analysis is much less conservative, because of the offline-to-online relative performance measure, respecting the fairness in performance evaluation under different input sequences. However, it is not easy to identify the worst-case relative performance and develop online algorithms with the best CR, as we frequently encounter nonconvex and combinatorial problems at first sight. In this paper, we shall tackle these challenges and address the online storage-assisted peak-reduction maximization problem. Following are our contributions.

⊳ In Section 3, we rigorously define the problem of maximizing the peak-demand reduction by utilizing limited energy storage. Moreover, we uncover the inherent structural insights into the optimal offline solution derived with perfect information.

⊳ In Section 4, we design the first and optimal online algorithm in the sense that it achieves the best CR among all online algorithms for the peak-reduction maximization problem. The algorithm chooses the action in each round to maintain an offline-to-online ratio to be no more than the best CR. Notably, we show that we can obtain the best CR in polynomial time by solving a linear number of linear-fractional programs.

⊳ In Section 5, we extend our approach to design an adaptive online algorithm that achieves adaptively-improved average-case performance while retains the same CR, i.e., the optimal worst-case performance. The underlying idea is that the revealed input at each round prunes the uncertainty set and enables us to respond to the non-occurrence of previously identified worst-case input sequences. Thus, we can adaptively pursue a possibly better offline-to-online ratio at each round for improved average-case performance, instead of constantly maintaining the best CR.

⊳ In Section 6, we conduct simulations on real-world traces and evaluate the empirical performance of our algorithms in representative scenarios. Simulation results show that our online algorithms improve the peak-demand reduction by more than 20% as compared to baseline alternatives. Further, with little future information, they achieve up to 81% of the (best possible) peak reduction attained by the optimal offline algorithm with perfection information.

Due to the page limitation, omitted proofs can be found in our technical report [24].

2 RELATED WORK

Using energy storage. As mentioned in the Introduction, more customers exploit energy storage for cost-savings on their electric bills and possible arbitrage opportunities. This phenomenon raises researchers' interests along this line, as exemplified below. Ref. [36] assessed the storage value for energy arbitrage and regulation services in New York. Ref. [35] explored time-average cost reduction opportunities of using existing storage, like UPS units of data centers. Another kind of existing storage refers to electric vehicles, and the economics of vehicle-to-grid services has been examined in [37]. The storage is valuable not only for commercial consumers, but also for residential customers, as discussed in [17, 22] and [38]. In addition to cost-saving, ref. [16] investigated reducing emission footprint of the power grid using distributed energy storage. In short, researchers are exploring the applications and potential financial interests of storage from various aspects.

Peak-demand charge. The peak-demand charge has attracted attention from both utilities and large-load customers. Ref. [26] analyzes the adoption of demand charge given the competition of isolated industrial customers. Ref. [42] considered an extended peak-demand charge which relates to the largest accumulated consumption over several periods. In the literature, researchers have studied multiple approaches for utilities and large-load customers in response to the peak-demand charge. Based on day-ahead load predictions, ref. [31] studies the joint optimization of peak shaving and frequency regulation with energy storage. Under a peak-based pricing model, ref. [43] considers the online economic dispatching of local generators in microgrids. Ref. [44] and ref. [18] respectively examine the scheduling and the pricing of electric vehicle services under the electricity tariff with peak-demand charge. See a survey in [34] for more examples. Overall, the peak-demand charge brings utilities and consumers for efficient and reliable power systems.

In our paper, we consider that a large-load consumer utilizes his or her energy storage to reduce the peak-demand charge. We focus on maximizing the peak-demand reduction brought by using energy storage. In contrast with directly considering the peak demand in a minimization problem, we suppose that the reduction in the maximization problem emphasizes more on the benefit brought by the energy storage and provides a fairer comparison on the efficacy of algorithms under different uncertainty sets or demand profiles, which will be made more clear as we proceed. In addition, interested readers can find preliminary results on the peak-demand minimization counterpart in [25] and [23].

Online competitive analysis. Online competitive analysis has been useful in electric vehicle charging [33, 41, 44] and economic dispatching [9, 43]. The competitive analyses of cost minimization and utility maximization have different challenges [30]. Although we can easily transform the problem from one to the other in the offline scenario, solutions and results for one problem may not directly apply to its counterpart in the online setting. We herein consider online peak-demand reduction maximization.

We adopt a similar algorithmic framework to that of [14, 20, 41, 44, 45] for the proposed online problem. The resulting algorithms are parameterized by a ratio "pursued" in each decision-making round. Like model predictive control, the algorithms make the decision of each round with the help of an offline optimization problem. It turns out that our algorithms are effective and efficient from theoretical analysis and empirical validations.

3 PROBLEM FORMULATION

In this section, we elaborate on the model of applying energy storage to reduce the peak-demand. We formulate a peak-demand reduction maximization problem under a capacity constraint and discharging rate constraint. We also analyze the optimal offline solution to the problem given that the entire input sequence is known, laying foundations for our focus on the practical online scenario. In Table 1, we summarize several useful symbols.

Table 1: A summary of symbols.

Symbols	Meanings
c	scaled storage energy capacity (kWh)
T	number of time slots
$\boldsymbol{d}$	a demand profile, $\boldsymbol{d} = [d_1, d_2, \cdots, d_T]' \in \mathbb{R}^T$, where d_t is the net demand at time slot t (kWh)
$\underline{d}, \overline{d}$	lower and upper bounds of the net demand at a time slot (kWh)
δ_t	discharge amount at time slot t, $\delta_t \geq 0$ (kWh)
$\bar{\delta}$	maximum discharge amount per time slot (kWh)
$v(\boldsymbol{d})$	peak usage of the optimal offline solution under the demand profile $\boldsymbol{d}$
$\sigma(\boldsymbol{d})$	peak-demand reduction of the optimal offline solution under the demand profile $\boldsymbol{d}$
$\boldsymbol{\delta}(\pi, \boldsymbol{d})$	discharging vector of pCR-PRM(π) under the demand profile $\boldsymbol{d}$
π^*	the best possible CR among all online algorithms
π_t^*	adaptive CR at time slot $t \in [T]$
$[n]$	the set $\{1, 2, \ldots, n\}$ for $n \in \mathbb{N}$, wherein $[0]$ reduces to the empty set
$[x]^+$	the maximum of 0 and $x \in \mathbb{R}$
$\mathcal{X} \backslash \mathcal{Y}$	$\{x \mid x \in \mathcal{X}$ & $x \notin \mathcal{Y}\}$ for two sets $\mathcal{X}$ and $\mathcal{Y}$

3.1 Mathematical Model

We assume that the peak demand of a large-power customer may occur in T time slots of an on-peak period, like from 9 a.m. to 10 p.m. for a shopping mall [19]. Each slot corresponds to 15 or 30 minutes by the power measurement for the peak-demand charge. Knowing the slot duration, we can convert the average power demand (in kW) into the consumed energy (in kWh) for each time slot.

Net electricity demand: We consider a general setting where the excess renewable generation cannot recharge the storage system. The self-owned renewable generations are not sufficient to cover the gross demand of the large-load customer during the T slots. Then, we use $\boldsymbol{d} \in \mathbb{R}^T$ to denote the net demand profile, which is given by the difference between the gross demand and the renewable generation. Although the cost of renewable generation is negligible, the volatile renewable generations exacerbate the unpredictability of net demands over time. Consequently, it is hard to estimate d_t accurately in previous time slots. We just know the lower and upper bounds of the net demand in a future time slot. For brevity, we herein focus on the case with uniform bounds, namely $d_t \in [\underline{d}, \overline{d}]$ for all $t \in [T]$, while our approach also applies to the case with time-variant bounds after slight modifications.

Energy storage: Let c (in kWh) be the energy storage capacity scaled by a factor concerning the maximum depth of discharge and the discharging efficiency. Let $\bar{\delta}$ (in kWh) denote the maximum discharge amount per time slot. In practice, we can hybridize different energy storage technologies with complementary characteristics [13]. For example, flywheel and supercapacity are high power devices, while fuel cells and pumped hydro are high energy devices. In this way, we can attain desired c and $\bar{\delta}$ to meet long-term energy needs and short-term power needs. We consider the scenario that we have fully charged the energy storage before the on-peak period and discharge the storage in the on-peak duration to reduce the peak-demand. We denote the discharging vector by $\boldsymbol{\delta} = [\delta_1, \delta_2, \cdots, \delta_T]' \in \mathbb{R}^T$, where δ_t (in kWh) is the discharge amount at time slot t. Then, the characteristics of the storage system leads to the capacity constraint $\sum_{t=1}^{T} \delta_t \leq c$ and the rate constraints $\delta_t \leq \bar{\delta}$, for all $t \in [T]$.

3.2 Problem Formulation

To investigate the benefit of energy storage, we formulate and study the following peak-demand reduction maximization (PRM) problem with a capacity constraint and discharging rate constraint,

$$\textbf{PRM:} \max_{\boldsymbol{\delta}\in\mathbb{R}^T} \max_{t\in[T]} d_t - \max_{t\in[T]} (d_t - \delta_t)$$
$$\text{subject to } \sum_{t=1}^{T} \delta_t \le c; \tag{1}$$
$$0 \le \delta_t \le \min\{\bar{\delta}, d_t\}, \text{ for all } t \in [T].$$

The objective represents the peak-demand reduction introduced by the energy storage. If there is no storage, the peak usage under the demand profile $\boldsymbol{d}$ is $\max_{t\in[T]} d_t$. After applying the discharging vector $\boldsymbol{\delta}$, we reduce the peak usage to $\max_{t\in[T]} (d_t - \delta_t)$. The discharging vector should satisfy the capacity and discharging rate constraints. Moreover, each discharge amount should not exceed the demand of the corresponding slot. Our goal is to maximize the peak-demand reduction by using energy storage, directly relating to the cost reduction on the peak-demand charge. Most existing studies consider the peak minimization objective, namely, $\min_{\boldsymbol{\delta}} \max_{t\in[T]} (d_t - \delta_t)$. In contrast, we adopt the peak-demand reduction maximization objective in PRM, because it makes us focus on the benefit brought by the energy storage. We observe that the two objectives are consistent in the offline setting because their summation remains a known constant $\max_{t\in[T]} d_t$ under any feasible solution. Notwithstanding, we should differentiate them in the online setting where we lack the information of $\max_{t\in[T]} d_t$, let alone we evaluate the online algorithms by competitive ratios. Moreover, in the online setting, we consider the performance guarantee over an uncertainty set of the demand profiles. It is clear that $\max_{t\in[T]} d_t$ is non-constant among different demand profiles. Considering a peak-reduction value relative to $\max_{t\in[T]} d_t$ instead of the absolute peak-demand value $\max_{t\in[T]} (d_t - \delta_t)$ provides a fairer performance comparison among different demand profiles. To the best of our knowledge, our work is the first to study the peak-demand reduction maximization by competitive analysis.

3.3 Optimal Offline Solution

Clearly, when the demand profile $\boldsymbol{d}$ is known, we can easily solve the PRM problem by linear programming. Moreover, the following proposition states that the optimal offline solution to PRM presents a particular threshold-based structure.

Proposition 1. *Given a demand profile $\boldsymbol{d} \in \mathbb{R}^T$, there exists $v \in \mathbb{R}$ with $\sum_{t=1}^{T} [d_t - v]^+ = c$ and an optimal solution to PRM is given by*

$$\delta_t^* = \left[d_t - \left[\max_{t\in[T]} \left[d_t - \bar{\delta} - v \right]^+ + v \right]^+ \right]^+, \text{ for all } t \in [T].$$

Note that $[x]^+ \triangleq \max\{x, 0\}$. A useful observation is that the optimal solution to PRMis also optimal for the peak-demand minimization problem with the same constraints. For notational convenience, we respectively use $v(\boldsymbol{d})$ and $\sigma(\boldsymbol{d})$ to denote the optimal peak usage and the optimal peak-demand reduction after discharging the stored energy, namely,

$$v(\boldsymbol{d}) = \max_{t\in[T]} (d_t - \delta_t^*) \text{ and } \sigma(\boldsymbol{d}) = \max_{t\in[T]} d_t - \max_{t\in[T]} (d_t - \delta_t^*),$$

where $\boldsymbol{\delta}^*$ is the optimal solution to PRM. While solving the offline PRM problem is easy, it is challenging to determine the discharge amounts in real-time without knowing future demands. Ref. [2] proposed an optimal competitive online algorithm for the peak minimization, assuming that the original peak usage $\max_{t\in[T]} d_t$ is known a priori. However, such an assumption is far from practical. In this paper, we consider a more general model where the prior information on the peak usage comprises the lower and upper demand bounds only. Also, we propose, in the next section, an optimal competitive algorithm for the online PRM problem. Overall, our work complements the literature by considering the peak-demand reduction maximization and not assuming to know the exact value of the original peak usage.

4 ONLINE ALGORITHM

In this section, we shall propose the first competitive online algorithm for PRM. This algorithm is parameterized by the best possible CR among all online algorithms and attains the optimal worst-case relative performance guarantee regarding the offline-to-online ratio of peak-demand reduction. Specifically, our optimal algorithm makes the decision of each round by maintaining an offline-to-online ratio to be no more than the best CR. Clearly, the best possible CR varies as we change the time-slot number (T), the demand bounds ($\underline{d}$ and $\overline{d}$), the storage capacity (c), and the maximum discharging limit ($\bar{\delta}$). In general, computing the best CR involves hard min-max optimization and we have to resort to dynamic programming which is time-consuming and computationally expensive. Fortunately, as a unique technical contribution, we show that we can obtain the best CR for the online PRM by solving a linear number of linear-fractional programs in parallel. More details and insights will be presented later.

Recall that the best possible CR essentially captures the price of uncertainty before we know any inputs, while the price of uncertainty may change as we observe more data and implement actions in past time slots. In practice, we intend to adaptively exploit the real-time information and pursue the updated prices of uncertainty. The particular structure of the proposed algorithm enables us to achieve this purpose and thus get improved average-case performance. We shall elaborate on this extension in the next section.

4.1 Online Setting

In the online setting, we do not know the exact value of d_t until time slot t. The empirical information only tells us that $d_t \in [\underline{d}, \overline{d}]$, for all $t \in [T]$. We denote the set of all possible demand profiles by $\mathcal{D} = \{\boldsymbol{d} \in \mathbb{R}^T \mid \underline{d} \le d_t \le \overline{d}, \forall t \in [T]\}$. Other prior information includes the storage capacity (c), the maximum discharging limit $\bar{\delta}$, and the time-slot number (T). We present, in Fig. 2, an illustrative flowchart of an online algorithm for PRM.

As mentioned in the introduction, concerning robustness and fairness, CR is a proper measure in online optimization with given resources [4]. It quantifies the relative performance between an online algorithm and the optimal offline benchmark. For a maximization problem like PRM, the CR of a deterministic online algorithm $\mathfrak{A}$ is defined as the largest ratio between the objective value under an optimal offline solution and that attained by the algorithm over all possible input sequences (e.g., the demand profiles in PRM), namely,

$$CR_{\mathfrak{A}} = \max_{\boldsymbol{d}\in\mathcal{D}} \frac{\sigma(\boldsymbol{d})}{\sigma_{\mathfrak{A}}(\boldsymbol{d})},$$

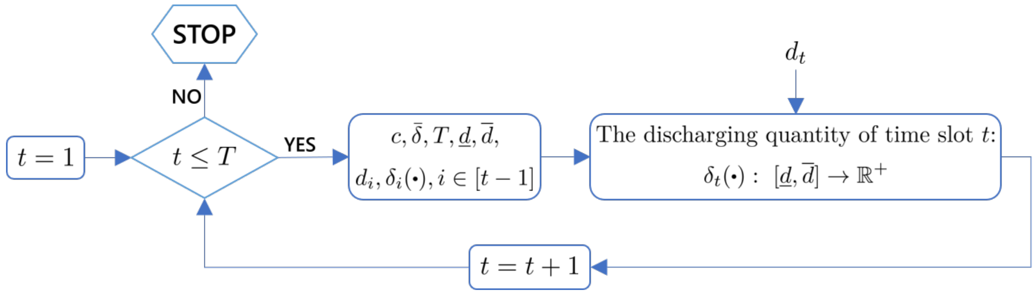


Figure 2: A flowchart of an online algorithm for PRM.

where $\sigma_{\mathfrak{A}}(\boldsymbol{d})$ refers to the objective values of PRM under the online algorithm and the input sequence $\boldsymbol{d}$. For a randomized algorithm $\mathfrak{A}$, we can extend the concept of CR by replacing $v_{\mathfrak{A}}(\boldsymbol{d})$ with $\mathrm{E}[v_{\mathfrak{A}}(\boldsymbol{d})]$, where the expectation is due to the random strategies of the algorithm $\mathfrak{A}$. We always have $CR_{\mathfrak{A}} \geq 1$ and a smaller CR indicates that the algorithm can perform more closely to the optimal offline case with perfect information. We expect to find the online algorithm with the smallest CR, which is the best possible CR among all online algorithms. The best CR not only verifies the optimal performance of the online algorithm, but also quantifies the essential cost of not knowing the future. In other words, the best CR captures the price of uncertainty in the considered online problem.

In the following, we show that it suffices to focus on deterministic online algorithms for the best CR regarding the online PRM.

Proposition 2. *For any randomized algorithm $\mathfrak{A}$, there exists a deterministic online algorithm $\mathfrak{B}$ such that $\mathrm{E}[\sigma_{\mathfrak{A}}(\boldsymbol{d})] \leq \sigma_{\mathfrak{B}}(\boldsymbol{d})$ over all possible input sequence $\boldsymbol{d} \in \mathcal{D}$.*

4.2 Overview of the CR-Pursuit framework

The usual practice is to propose an online algorithm and then compute its CR for evaluating the performance. In contrast, we herein ask whether there exists an online algorithm with given CR. This question relates to a useful framework for designing competitive online algorithms, called CR-Pursuit. As the name indicates, the CR-Pursuit algorithmic framework requires us to sequentially make online decisions by "pursuing" a prescribed CR. For a maximization problem and a given CR, each CR-Pursuit algorithm will choose actions to maintain an offline-to-online objective ratio to be no more than the CR, in each decision-making round. Despite the conciseness of the idea, there is no general recipe for pursuing the given CR. The means of designing algorithms under CR-Pursuit is problem-specific, as exemplified in [20] and [41].

We herein summarize several useful observations regarding the CR-Pursuit framework. First, to maintain the offline-to-online ratio in each decision-making round, we usually need to solve an offline problem according to the observed inputs and implemented actions so far. If the offline version of a considered problem is easier, then it is less challenging to design algorithms under CR-Pursuit. Second, the CR-Pursuit framework generally generates a family of algorithms, each of which is characterized by a specific CR. Clearly, the best algorithm among these is the one with the smallest CR. We wonder whether the best algorithm under CR-Pursuit is optimal among all online algorithms. If so, then the CR-Pursuit framework greatly reduces the search space of optimal online algorithms. Overall, there are three main challenges in designing CR-Pursuit algorithms: finding a proper way to sequentially maintain a given CR, identifying the best CR which can be pursued, and checking whether the best CR under CR-Pursuit is the optimal one among all online algorithms. In the following, we shall tackle these challenges with the online PRM. First of all, we shall devise a collection of online algorithms under the CR-Pursuit framework. Then, we show that the best algorithm among these is also optimal among all online algorithms for PRM. Finally, we attain the best online algorithm for PRM by finding the best possible CR.

Algorithm 1: pCR-PRM(π)

for $t = 1, 2, \ldots, T$ **do**
  The discharge amount at time slot t is given by $\delta_t(\pi, \boldsymbol{d}) = [d_t - \max_{k \in [t]} d_k + \sigma(\boldsymbol{d}^t)/\pi]^+$;

4.3 Optimal Online Algorithm for PRM

Without knowing future inputs, it seems impossible to maintain the offline-to-online objective ratio to be no more than a given ratio. If we do not know the time-slot number (T), we can simply assume that there are no future inputs, as has been done in the existing results [20, 41]. However, in the online PRM, we are challenged by the facts that T is known and the future inputs will affect the online and optimal offline peak-demand reductions. As a result, we shall introduce a reference input and adaptively update it at each time slot $t \in [T]$. Specifically, we set the reference input sequence at time slot t as

$$\boldsymbol{d}^t = [d_1 \; d_2 \; \cdots \; d_t \; \underline{d} \; \cdots \; \underline{d}]'.$$

Clearly, the reference input sequence is a combination of the observed demands until time slot t and the lowest possible future demands, indicating the most optimistic forecast. Given a ratio π, we shall apply CR-Pursuit framework and maintain the offline-to-online ratio under the reference input sequence $\boldsymbol{d}^t$ to be more than π at each time slot $t \in [T]$. To this end, we design a family of algorithms, each of which is characterized by a prescribed ratio π and called pCR-PRM(π). The details of pCR-PRM(π) are given in Algorithm 1. For notational convenience, we use $\boldsymbol{\delta}(\pi, \boldsymbol{d})$ to denote the output sequence of pCR-PRM(π) under an input sequence $\boldsymbol{d}$.

Alert readers may notice two issues. First, pCR-PRM(π) may generate infeasible solutions to PRM. Second, we wonder whether the formula in Algorithm 1 can maintain the offline-to-online objective ratio under $\boldsymbol{d}^t$ to be no more than π, for all $t \in [T]$. For the first issue, we give the following definition in terms of feasibility of pCR-PRM algorithms.

Definition 3. *pCR-PRM(π) is feasible if, for any $\boldsymbol{d} \in \mathcal{D}$, the solution $\boldsymbol{\delta}(\pi, \boldsymbol{d})$ is feasible for PRM.*

We address the second issue by the following lemma.

Lemma 4. *Given pCR-PRM(π), it holds for any $\boldsymbol{d} \in \mathcal{D}$ that*

$$\max_{k\in[t]} d_k - \max_{k\in[t]} (d_k - \delta_k(\pi, \boldsymbol{d})) \geq \sigma(\boldsymbol{d}^t)/\pi, \text{ for all } t \in [T].$$

Therefore, we conclude that pCR-PRM(π) can maintain the given CR π if and only if it is feasible. Identifying the best pCR-PRMalgorithm is equivalent to finding the smallest π such that pCR-PRM(π) is feasible. To this end, we shall first characterize the set of all ratios such that the corresponding pCR-PRM algorithms are feasible. For this purpose, we define an inventory function, which maps a given ratio π to the maximum accumulated discharge amount over all possible demand profiles under pCR-PRM(π):

$$\Phi(\pi) = \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t=1}^{T} \delta_t(\pi, \boldsymbol{d}).$$

We show below that the feasibility of pCR-PRM(π) is mainly subject to the capacity constraint.

Proposition 5. pCR-PRM(π) *is feasible if and only if* $\Phi(\pi) \leq c$.

The following lemma unravels the monotonicity of $\Phi(\pi)$.

Lemma 6. *The function $\Phi(\pi)$ is non-increasing and strictly decreases in π when $\Phi(\pi) > 0$.*

From Lemma 6, we see that there exists a ratio $\bar{\pi} > 1$ such that pCR-PRM(π) is feasible only if $\pi \geq \bar{\pi}$. We shall show that pCR-PRM($\bar{\pi}$) is the best pCR-PRM algorithm. More importantly, the best pCR-PRM algorithm also attains the optimal CR among all online algorithms for PRM, as stated in the following theorem.

Theorem 7. *Given c, $\bar{\delta}$, T, and $\mathcal{D}$, the unique solution π^* to the equation $\Phi(\pi) = c$ is the best possible competitive ratio among all online algorithms for PRM.*

The above theorem indicates that the solution to $\Phi(\pi) = c$ characterizes the price of uncertainty regarding the online PRM. To find the best online algorithm for PRM, it suffices to search for the best feasible pCR-PRM algorithm. In the next subsection, we shall show an efficient way to find the best possible CR π^*.

4.4 Finding the Optimal Competitive Ratio π^*

Deriving the analytic expression of π^* over parameters $(c, \bar{\delta}, T, \underline{d}, \overline{d})$ is attractive but challenging. Instead, we shall explore efficient numerical methods for the best CR π^*. As a unique technical contribution, we show that computing π^* is not much harder than linear programming. Specifically, we first show that the best CR π^* is the maximum of an exponential number of linear-fractional programs. Then, we exploit the problem structure and show that only a linear number of such programs are necessary in terms of the time-slot number T. Note that we can transform each involved linear-fractional program into an equivalent linear program by the techniques in [5, Section 4.3.2].

Recall the key formula and the feasibility condition ($\Phi(\pi) \leq c$) of pCR-PRM(π). We see that pCR-PRM(π) is feasible if and only if for any nonempty subset $\mathcal{I}$ of the index set $[T]$ and any $\boldsymbol{d} \in \mathcal{D}$, it holds that $\sum_{i\in\mathcal{I}} [d_i - \max_{k\in[i]} d_k + \sigma(\boldsymbol{d}^i)/\pi]^+ \leq c$. It follows that

$$\frac{\sum_{i\in\mathcal{I}} \sigma(\boldsymbol{d}^i)}{c + \sum_{i\in\mathcal{I}} \left(\max_{k\in[i]} d_k - d_i\right)} \leq \pi. \tag{2}$$

A worst-case input sequence for pCR-PRM(π^*) refers to a demand profile $\boldsymbol{d} \in \mathcal{D}$, under which the pCR-PRM(π^*) algorithm will use up the storage, namely $\sum_{t=1}^{T} \delta(\pi^*, \boldsymbol{d}) = c$. Since $\Phi(\pi^*) = c$, there exists a worst-case input sequence for pCR-PRM(π^*). Let $\boldsymbol{d}^*$ be a worst-case input sequence and $\mathcal{I}^*$ be the set $\{i \in [T] \mid \delta_t(\pi^*, \boldsymbol{d}^*) > 0\}$. Then, we observe that the equality in Formula (2) holds under the index set $\mathcal{I}^*$ and the input sequence $\boldsymbol{d}^*$. Therefore, we attain the following proposition.

Proposition 8. *Given c, $\bar{\delta}$, T, and $\mathcal{D}$, the best possible CR for the online PRM is given by*

$$\pi^* = \max_{\mathcal{I}\subseteq[T], \boldsymbol{d}\in\mathcal{D}} \frac{\sum_{i\in\mathcal{I}} \sigma(\boldsymbol{d}^i)}{c + \sum_{i\in\mathcal{I}} \left(\max_{k\in[i]} d_k - d_i\right)}.$$

Proposition 8 follows directly from Lemma 6 and Proposition 5. Equipped with Proposition 8, we shall transform the computation of the best CR π^* into solving a sequence of linear-fractional programs. Before proceeding, we observe that $\sigma(\boldsymbol{d}) = \max_{t\in[T]} d_t - v(\boldsymbol{d})$ and the largest element in $\boldsymbol{d}^t$ appears in the first t elements. Moreover, we introduce a useful lemma below.

Lemma 9. *Given $x > y > 0$ and $z > 0$, it follows that $\frac{x+z}{y+z} \leq \frac{x}{y}$.*

Next, we shall define a family of linear-fractional programs. Each program is parameterized by a nonempty set $\mathcal{I} \subseteq [T]$ and its optimal objective value gives a lower bound of the best CR π^*:

$$\begin{aligned}
\text{CR-Comp}(\mathcal{I}):\ & \max_{\boldsymbol{d}\in\mathcal{D}, u_i, \delta_{ij}} \frac{\sum_{i\in\mathcal{I}} (m_i - u_i)}{c + \sum_{i\in\mathcal{I}} (m_i - d_i)} \\
\text{subject to } & \sum_{j=1}^{T} \delta_{ij} \leq c, \text{ for all } i \in [T]; \\
& 0 \leq \delta_{ij} \leq \bar{\delta}, \text{ for all } i, j \in [T]; \\
& d_j - \delta_{ij} \leq u_i, \text{ for all } 1 \leq j \leq i \leq T; \\
& \underline{d} - \delta_{ij} \leq u_i, \text{ for all } 1 \leq i < j \leq T; \\
& d_k \leq m_i, \text{ for all } k \in [i] \text{ and } i \in \mathcal{I}.
\end{aligned}$$

Now, let us interpret the variables, constraints and objective of CR-Comp($\mathcal{I}$). Auxiliary variables m_i, $i \in [T]$ are related to $\max_{k\in[i]} d_k$. More specifically, by Lemma 9 and the constraints on the last line, CR-Comp($\mathcal{I}$) will attain its optimum when $m_i = \max_{k\in[i]} d_k$, for all $i \in \mathcal{I}$. The remaining constraints of CR-Comp($\mathcal{I}$) are associated with the offline PRM problem solved at each time slot under a pCR-PRM algorithm. Precisely, for each $i \in [T]$, the variable v_i and the variables δ_{ij}, $j \in [T]$ are respectively related to the optimal objective value and the optimal solution to PRM under the demand profile $\boldsymbol{d}^i$. CR-Comp($\mathcal{I}$) will attain its optimum

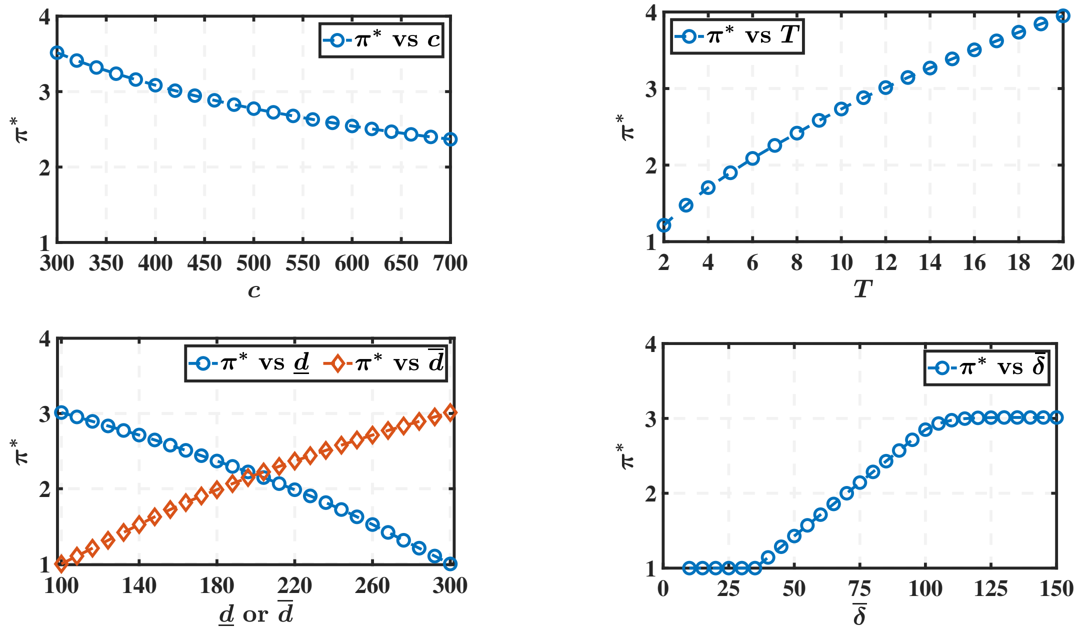


Figure 3: The best CR π^* varies as the parameters $(c, \bar{\delta}, T, \underline{d}, \overline{d})$. Default values are $c = 420$, $T = 12$, $\underline{d} = 100$, $\overline{d} = 300$, and $\bar{\delta} = 300$.

when $v_i = v(\boldsymbol{d}^i)$, for all $i \in \mathcal{I}$. Moreover, the objective function of CR-Comp$(\mathcal{I})$ corresponds to the left part of Formula (2), noting that $\sigma(\boldsymbol{d}^i) = \max_{k\in[i]} d_k - v(\boldsymbol{d}^i)$. With a slight abuse of notation, we also use CR-Comp$(\mathcal{I})$ to denote the optimal objective value of the linear-fractional program. As a whole, by Lemma 9, Proposition 8, and the above analysis, we attain the following proposition.

Proposition 10. *For each nonempty set $\mathcal{I} \subseteq [T]$, it holds that*

$$\textit{CR-Comp}(\mathcal{I}) = \max_{\boldsymbol{d}\in\mathcal{D}} \frac{\sum_{i\in\mathcal{I}} \sigma(\boldsymbol{d}^i)}{c + \sum_{i\in\mathcal{I}} \left(\max_{k\in[i]} d_k - d_i\right)}.$$

Thus, by Proposition 8 and Proposition 10, we conclude that the best possible CR π^* equals $\max_{\mathcal{I}\subset[T]}$ CR-Comp$(\mathcal{I})$. That is to say, we can compute π^* by solving a collection of linear-fractional programs CR-Comp$(\mathcal{I})$. Recall that we can convert CR-Comp$(\mathcal{I})$ into an equivalent linear program. Thus, computing π^* is not much harder than solving a collection of linear programs. Moreover, we can compute these programs in parallel, since none of the CR-Comp$(\mathcal{I})$ programs relies on the solution to another. Nevertheless, a direct application of the above results requires to solve an exponential number of linear programs, which is undesirable. Thus, we are motivated to exploit the structure of pCR-PRM(π^*) and exclude as many redundant programs as possible. To this end, we derive the following lemma. It identifies a particular worst-case input sequence for pCR-PRM(π^*), which continuously discharge stored energy until using up the capacity.

Lemma 11. *There exists a worst-case input sequence $\boldsymbol{d} \in \mathcal{D}$ for pCR-PRM(π^*) such that $\delta_t(\pi^*, \boldsymbol{d}) = 0$ if t is greater than a certain index and $\delta_t(\pi^*, \boldsymbol{d}) > 0$ otherwise.*

By Proposition 8, Proposition 10, and Lemma 11, we see that there exists $k \in [T]$ such that $\pi^* =$ CR-Comp$([k])$. Thus ,we have the following theorem stating that only a linear number (T) of CR-Comp$(\mathcal{I})$ programs are necessary for the best possible CR π^*, instead of the exponential number 2^T.

Theorem 12. *The best possible competitive ratio π^* for the online PRM is given by*

$$\pi^* = \max_{\mathcal{I}\in\{[t]|t\in[T]\}} \textit{CR-Comp}(\mathcal{I}).$$

In Fig. 3, we illustrate how the best possible CR π^* varies as each of the parameters $(c, \bar{\delta}, T, \underline{d}, \overline{d})$ changes. We observe that the optimal CR π^* increases as T increases and c decreases. π^* becomes smaller when $\underline{d}$ and $\overline{d}$ get closer as the uncertainty set is shrinked. Moreover, the best CR π^* is non-decreasing in $\bar{\delta}$. Particularly, when $\bar{\delta}$ is small enough, e.g., $\bar{\delta} \le c/T$, the best reduction for both online and offline is $\bar{\delta}$, and the optimal CR is one. Contrary to this, when $\bar{\delta}$ is large enough, the rate constraint will not be active in the offline problem to be solved in each round of pCR-PRM(π^*) under the worst-case input sequence; consequently, the best CR π^* remains constant when $\bar{\delta}$ increases.

5 ADAPTIVE PCR-PRM ALGORITHM

While pCR-PRM(π^*) attains the optimal CR among all online algorithms for PRM, it merely focuses on the worst-case performance, which may restrict its performance in practice. We herein extend the pCR-PRM(π^*) algorithm by adaptively exploiting the revealed information of previous slots. Here is the intuitive idea: when we realize from the observed inputs that the net demand profile is by no means a worst-case input sequence, we should be more opportunistic and attempt to maintain smaller ratios in the following time slots. In this way, we can improve the average-case performance and still attain the optimal worst-case performance. The underlying reason lies in that the price of future uncertainty changes as we

Algorithm 2: Adaptive pCR-PRM Algorithm

for $t = 1, 2, \ldots, T$ **do**
  Obtain π_t^* according to Algorithm 3, The discharge amount at time slot t is given by

$$\delta_t(\pi, \boldsymbol{d}) = \left[d_t - \max_{k\in[t]} d_k + \sigma(\boldsymbol{d}^t)/\pi_t^*\right]^+$$

Algorithm 3: A Bisection Method for π_t^* in the Adaptive pCR-PRM Algorithm

Input: $c, T, \underline{d}, \overline{d}$, observed inputs $d_k, k \in [t]$, implemented actions $\delta_k, k \in [t-1]$, and π_{t-1}^*;
Output: The adaptive CR at time slot t under the adaptive pCR-PRM: π_t^*;
$\pi_{lb} = \pi_t^{lb}, \pi_{ub} = \pi_{t-1}^*$,
$q = \max_{\mathcal{I}\in\mathcal{I}_t}$ AdaCR-Threshold$(\pi_{lb}, \mathcal{I})$;
if $q \le (c - \sum_{k=1}^{t-1}\delta_k)$ **then**
  $\pi_t^* = \pi_{lb}$,
  return
while $\pi_{ub} - \pi_{lb} \ge \epsilon$ **do**
  $\pi = (\pi_{lb} + \pi_{ub})/2$,
  $q = \max_{\mathcal{I}\in\mathcal{I}_t}$ AdaCR-Threshold$(\pi, \mathcal{I})$;

$$\begin{cases}\pi_{lb} = \pi & \text{if } q > (c - \sum_{k=1}^{t-1}\delta_k);\\ \pi_{ub} = \pi & \text{otherwise;}\end{cases}$$

$\pi_t^* = \pi_{ub}$;

observe more inputs and capturing such variations is critical to the improvement of online decisions. Overall, the results in this section suggest the potential of merging efficiency into robustness.

With respect to the real-time information, we first extend the concept of CR to a time-variant adaptive CR for every online algorithm for PRM at each time slot t. Specifically, we make the following definition.

Definition 13. *Given revealed inputs $d_k, k \in [t]$, the adaptive CR at time slot t of an online algorithm $\mathfrak{A}$ is defined as*

$$\pi_t^{\mathfrak{A}} = \max_{\boldsymbol{x}\in\mathcal{D}\ \&\ x_k=d_k,\ for\ all\ k\in[t]} \frac{\sigma(\boldsymbol{x})}{\sigma_{\mathfrak{A}}(\boldsymbol{x})}.$$

Let $\mathcal{A}_t$ be the set of online algorithms whose first $(t-1)$ outputs are $\delta_k, k \in [t-1]$ given that the first $(t-1)$ inputs are $d_k, k \in [t-1]$. Then, considering the observed inputs $d_k, k \in [t]$ and implemented actions $\delta_k, k \in [t-1]$ so far, the best adaptive CR at time slot t is given by

$$\pi_t^* = \min_{\mathfrak{A}\in\mathcal{A}_t} \pi_t^{\mathfrak{A}}.$$

Similarly to before, the best adaptive CR at time slot t characterizes the price of uncertainty at time slot t, without knowing the demands of future time slots indexed by $k > t$. Specifically, an online algorithm can at best maintain the online-to-offline ratio of peak-demand reduction to be π_t^*, for all future inputs. It is clear that π_t^* is subject to the observed inputs and actions, for all $t \in [T]$. Based on the introduction of best adaptive CRs, we are ready to introduce the adaptive extension of pCR-PRM.

At each time slot t, the adaptive pCR-PRM maintains the online-to-offline ratio of the peak-demand reduction to be no more than the best adaptive CR π_t^*, instead of a constant CR π^*. We present the pseudocodes of the adaptive pCR-PRM in Algorithm 2. .

The remaining issue is on characterizing π_t^*, the best adaptive CR at each slot. To proceed, we rely on the following observations.

- We observe that the best adaptive CR π_1^* is no more than the best CR π^*, because we exploit the additional information d_1. Since the adaptive pCR-PRM algorithm makes decisions by pursuing the best adaptive CR at each time slot, we conclude that the sequence $\pi_t^*, t \in [T]$ is nonincreasing in t,
$$\pi_t^* \le \pi_{t-1}^*, \forall t \in [T].$$
- Given the online actions before time slot t, $\delta_k, k \in [t-1]$, the online peak-demand reduction under the demand profile $\boldsymbol{d}^t$ is no more than $\max_{k\in[t]} d_k - \max_{k\in[t-1]}(d_k - \delta_k)$. Thus, defining
$$\pi_t^{lb} \triangleq \frac{\sigma(\boldsymbol{d}^t)}{\max_{k\in[t]} d_k - \max_{k\in[t-1]}(d_k - \delta_k)},$$
we have $\pi_t^* \ge \pi_t^{lb}$.

Based on these observations, we shall show how to search for π_t^* by a bisection method. To this end, in the following, given observed inputs $d_k, k \in [t]$ and implemented decisions $\delta_k, k \in [t-1]$, we define a linear program parameterized by a ratio $\pi \in [\pi_{lb}^t, \pi_{t-1}^*]$ and a set $\mathcal{I} \in \mathcal{I}_t$, where $\mathcal{I}_t = \{[k]\backslash[t] \mid k = t, t+1, \ldots, T\}$:

$$\begin{aligned}\text{AdaCR-Threshold}(\pi, \mathcal{I}) : & [d_t - \max_{k\in[t]} d_k + \sigma(\boldsymbol{d}^t)/\pi]^+ \\ & + \max_{v_i, \delta_{ij}, d_i, i\in\mathcal{I}} \sum_{i\in\mathcal{I}} [d_i - m_i + (m_i - v_i)/\pi] \\ \text{subject to } & \sum_{j=1}^{T} \delta_{ij} = c, \text{ for all } i \in \mathcal{I}; \\ & 0 \le \delta_{ij} \le \bar{\delta}, \text{ for all } i \in \mathcal{I} \text{ and } j \in [T]; \\ & d_j - \delta_{ij} \le v_i, \text{ for all } i \in \mathcal{I} \text{ and } j \le i; \\ & \underline{d} - \delta_{ij} \le v_i, \text{ for all } t \le i < j \le T; \\ & d_k \le m_i, \text{ for all } k \in [i] \text{ and } i \in \mathcal{I}; \\ & \underline{d} \le d_i \le \overline{d}, \text{ for all } i \in \mathcal{I}.\mathcal{I}.\end{aligned}$$

The objective function corresponds to the sum of discharge amounts over a set of time slots assuming that we maintain the online-to-offline ratio of peak-demand reduction to be no more than π from the current time slot to T. Similar to CR-Comp$(\mathcal{I})$, the constraints of AdaCR-Threshold$(\pi, \mathcal{I})$ are due to the offline PRM problem solved in each time slot under the adaptive pCR-PRM. By similar arguments for computing the best CR π^*, we conclude that the adaptive CR at time slot t should be the smallest ratio π in $\left[\pi_{lb}^t, \pi_{t-1}^*\right]$ such that $\max_{\mathcal{I}\in\mathcal{I}_t}$ AdaCR-Threshold$(\pi, \mathcal{I})$ does not exceed the remaining inventory $c - \sum_{k=1}^{t-1}\delta_k$. Therefore, we can search for π_t^* by the bisection method clarified in Algorithm 3. Together with Algorithm 3, we complete the introduction of Algorithm 2 and now summarize the theoretical performance in the following proposition.

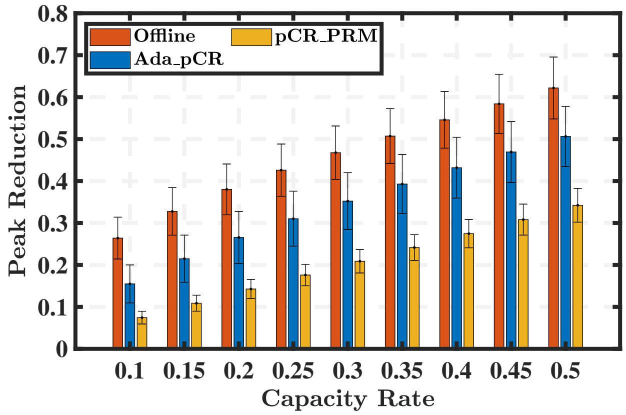


Figure 4: Peak-demand reduction under optimal offline solutions, pCR-PMD(π^*), and the adaptive pCR-PMD.

Proposition 14. *At each time slot t, Algorithm 2 achieves the optimal adaptive competitive ratio among all algorithms in $\mathcal{A}_t$.*

If the input sequence $\boldsymbol{d}$ is in the worst case regarding pCR-PRM(π^*), then $\pi_t^* = \pi^*$, for all $t \in [T]$. Otherwise, there is an index $\tau \in [T]$ such that $\pi_t^* < \pi^*$, for all $t \geq \tau$. From this perspective, we show that the adaptive pCR-PRM also attains the optimal CR among all online algorithms for PRM; moreover, it outperforms pCR-PRM(π^*) under general cases, as verified by the simulation results in the next section.

6 SIMULATION

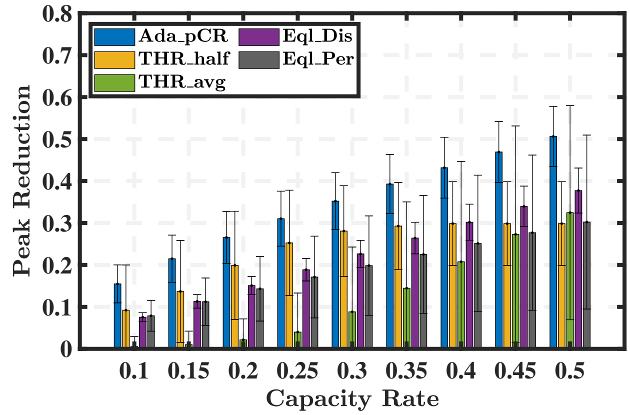
In this section, we apply the real-world traces to evaluate the performance of our algorithm under diverse settings. We also compare with conceivable alternatives. The results corroborate our theoretical findings and demonstrate the potentials for practical implementation of our approaches.

6.1 Simulation Setups

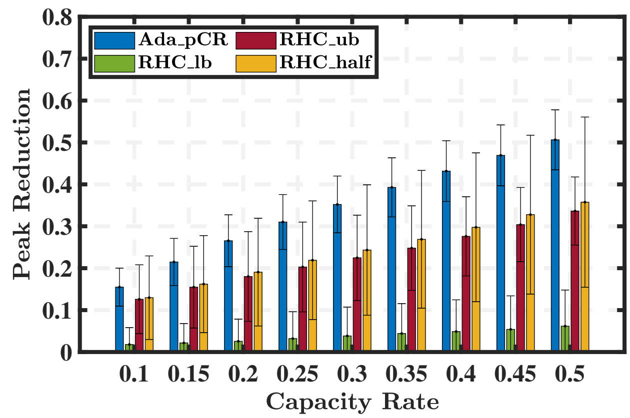
In the simulation, we consider a scenario where an EV charging station operator uses its storage to reduce its peak demand in a day. We obtain three-month electricity data from an EV charging station in Shenzhen, China. We divide the time into slots with 15-min length and derive the power demand of the charging station at each slot from the data. We then identify the on-peak duration in a day from the data. In particular, we consider a decision period of $T = 20$ slots and set the demand bounds as $(\underline{d}, \bar{d}) = (442.91, 1020.10)$ kWh, which are the minimum and maximum demand of the charging station in this three months, respectively.

6.2 Performance Evaluation

pCR-PRM(π^*) and Adaptive pCR-PRM We evaluate the performance of these two algorithms (*ref.* pCR_PRM and Ada_pCR) under different storage capacities and show the results in Fig. 4. The capacity rate represents the ratio between the storage capacity and the average daily demand of the charging station in the specified on-peak period. The peak reduction refers to the average peak reduction rate, which is the average ratio between the peak reduction



(a)



(b)

Figure 5: Empirical performance comparison as the storage capacity changes. a) Peak-demand reduction under four threshold-based algorithms and the adaptive pCR-PMD. b) Peak-demand reduction under three RHC algorithms and the adaptive pCR-PMD.

achieved by respective algorithms and the original peak demand. For Fig. 4, we observe that all the algorithms perform better as the capacity increases. Ada_pCR attains a higher peak reduction as compared with pCR_PRM, which corroborates our theoretical findings in Sec. 5. Our online algorithm adaptive pCR-PRM with little future information can achieve 59% ~ 81% of the peak reduction of the optimal offline solution derived with perfect information.

Comparison with Alternatives We mainly compare the adaptive pCR-PRM with two catalogs of conceivable alternatives, naive threshold-based algorithms and receding horizon control (RHC) algorithms. We show the results in Fig. 5a and Fig. 5b. In particular, we introduce the alternatives as follows,

(1) THR_half or THR_avg represents the algorithm discharging to a threshold at each slot until using up the storage. THR_half sets the threshold as $\left(\underline{d} + \bar{d}\right)/2$, and THR_avg sets the threshold as the average optimal offline peak demand after using the storage.

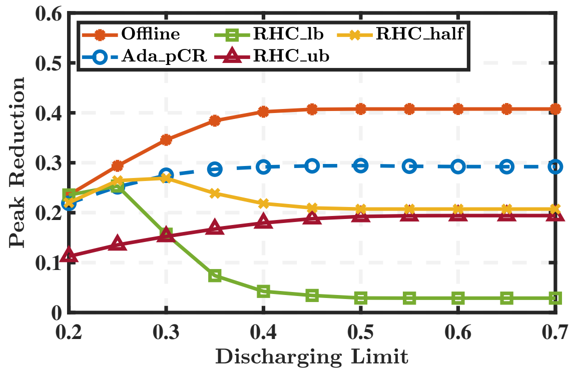


Figure 6: Effects of the maximum discharging rate.

(2) Eql_Dis equally distributes the energy storage capacity at each slot. Eql_Per discharges at a constant ratio of the demand at each slot until the capacity is running out, and sets the ratio the same as the capacity rate.
(3) RHC algorithms assume a look-ahead window of 5 slots (a quarter of the on-peak duration) in our simulation. At each slot, the RHC algorithms first compute the optimal solution based on the demand in the look-ahead window and their guesses beyond the window. Then, it implements the optimal solution at the current slot and recomputes the optimal at the next slot. RHC_lb, RHC_ub, and RHC_half assume the demand beyond the look-ahead window as $\underline{d}$, $\overline{d}$, and $\left(\underline{d}+\overline{d}\right)/2$, respectively.

From Fig. 5a, we observe that the adaptive pCR-PRM achieves the largest peak-shaving under different capacity rates, with at least 23% improvement against threshold-based algorithms. We observe from the Fig. 5b that the adaptive pCR-PRM has at least 20% improvement and a relatively small deviation on the peak reduction as compared to the RHC algorithms.

Impact of Discharging Limit We evaluate the performance of the adaptive pCR-PRM under different maximum charging rates and compare it with alternatives. The results are shown in Fig. 6. We normalize the discharging rate limit by the upper bound $\overline{d}$ of the demand at a slot. We observe that the adaptive pCR-PRM has close performance under different discharging limits, while the optimal offline achieves greater peak reduction as the discharging limit increases. Without knowing the future demand, the adaptive pCR-PRM with a larger charging rate limit may waste more energy at the previously observed peak demand which turns out not to be the peak in the whole period. This prevents the adaptive pCR-PRM from obtaining higher peak reduction at a larger discharging rate limit. Furthermore, the adaptive pCR-PRM outperforms other alternatives under different charging rate limits.

7 CONCLUSION

We study an online storage-assisted peak-reduction maximization problem. We focus on a scenario that a large-load power consumer with self-owned renewable generation uses energy storage to reduce its net peak demand during the on-peak period. We derive an optimal algorithm pCR-PRM(π^*) that achieves the best CR among all online algorithms. We obtain the best CR π^* by solving a linear number of linear-fractional programs. We further extend our algorithm to an adaptive one by exploiting observed inputs in real-time. The adaptive pCR-PRM achieves the best adaptive CR at each time slot given the revealed inputs and online decisions so far. It improves the average-case performance while maintaining the worst-case performance. Finally, We evaluate the empirical performance of our algorithms with real-world traces. We show that the adaptive pCR-PRM achieves close performance with the optimal offline solution with perfect information and outperforms conceivable alternatives. In the future, we shall consider energy storage systems with recharging, e.g., batteries and extend our techniques to more interesting scenarios like scheduling flexible loads [45].

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REFERENCES

[1] Hisham Alharbi and Kankar Bhattacharya. 2017. Stochastic optimal planning of battery energy storage systems for isolated microgrids. *IEEE Tran. Sustain. Energy* 9 (2017), 211–227.
[2] Amotz Bar-Noy, Matthew P Johnson, and Ou Liu. 2008. Peak shaving through resource buffering. In *International workshop on Approximation and Online Algorithms*. Springer, 147–159.
[3] Dimitris Bertsimas, David B. Brown, and Constantine Caramanis. 2011. Theory and applications of robust optimization. *SIAM Rev.* 53 (2011), 464–501.
[4] Allan Borodin and Ran El-Yaniv. 2005. *Online Computation and Competitive Analysis*. Cambridge University Press.
[5] Stephen Boyd and Lieven Vandenberghe. 2004. *Convex Optimization*. Cambridge University Press.
[6] Gwen Brown. 2017. Making sense of demand charges: What are they and how do they work? https://blog.aurorasolar.com/making-sense-of-demand-charges-what-are-they-and-how-do-they-work
[7] Niv Buchbinder, Shahar Chen, Joseph Seffi Naor, and Ohad Shamir. 2016. Unified Algorithms for Online Learning and Competitive Analysis. *Math. Oper. Res.* 41 (2016), 612–625.
[8] Aranya Chakrabortty and Marija D. Ilić. 2012. *Control and Optimization Methods for Electric Smart Grids*. Springer.
[9] Chi-Kin Chau, Guanglin Zhang, and Minghua Chen. 2016. Cost minimizing online algorithms for energy storage management with worst-case guarantee. *IEEE Trans. Smart Grid* 7 (2016), 2691–2702.
[10] CLP Power Hong Kong. 2020. Non-residential tariff of CLP. https://www.clp.com.hk/en/customer-service/tariff/business-and-other-customers/non-residential-tariff
[11] Clark W Gellings. 1985. The concept of demand-side management for electric utilities. *Proc. IEEE* 73 (1985), 1468–1470.
[12] Yuanxiong Guo and Yuguang Fang. 2013. Electricity cost saving strategy in data centers by using energy storage. *IEEE Trans. Parallel Distrib. Syst.* 24 (2013), 1149–1160.
[13] Reza Hemmati and Hedayat Saboori. 2016. Emergence of hybrid energy storage systems in renewable energy and transport applications – A review. *Renew. Sustain. Energy Rev.* 65 (2016), 11–23.
[14] Brady Hunsaker, Anton J. Kleywegt, Martin WP Savelsbergh, and Craig A. Tovey. 2003. Optimal online algorithms for minimax resource scheduling. 16 (2003), 555–590.
[15] Henrik Klinge Jacobsen and Sascha Thorsten Schröder. 2012. Curtailment of renewable generation: Economic optimality and incentives. *Energy Policy* 49 (2012), 663–675.
[16] Rishikesh Jha, Stephen Lee, Srinivasan Iyengar, Mohammad H. Hajiesmaili, David Irwin, and Prashant Shenoy. 2020. Emission-Aware Energy Storage Scheduling for a Greener Grid. In *Proceedings of the Eleventh ACM International Conference on Future Energy Systems (e-Energy '20)*. 363–373.
[17] Jason Leadbetter and Lukas Swan. 2012. Battery storage system for residential electricity peak demand shaving. *Energ. Buildings* 55 (2012), 685–692.

[18] Zachary J. Lee, John Z. F. Pang, and Steven H. Low. 2020. Pricing EV charging service with demand charge. *Electric Power Systems Research* 189 (2020), 106694.

[19] Mathilde Lepage. 2020. Dual day and night tariffs: When do they apply? https://www.energyprice.be/blog/2017/10/23/electricity-off-peak-hours/

[20] Qiulin Lin, Hanling Yi, John Pang, Minghua Chen, Adam Wierman, Michael Honig, and Yuanzhang Xiao. 2019. Competitive online optimization under inventory constraints. *Proc. SIGMETRICS* 3 (2019), 1–28.

[21] Zhenhua Liu, Adam Wierman, Yuan Chen, Benjamin Razon, and Niangjun Chen. 2013. Data center demand response: Avoiding the coincident peak via workload shifting and local generation. *Perform. Evaluation* 70 (2013), 770–791.

[22] Jingran Ma, S. Joe Qin, Timothy Salsbury, and Peng Xu. 2012. Demand reduction in building energy systems based on economic model predictive control. *Chem. Eng. Sci.* 67 (2012), 92–100.

[23] Yanfang Mo, Qiulin Lin, Minghua Chen, and S. Joe Qin. 2021. Online Peak-Demand Minimization Using Energy Storage. *ArXiv e-prints* (2021). arXiv:2103.00005.

[24] Yanfang Mo, Qiulin Lin, Minghua Chen, and S. Joe Qin. 2021. Optimal Online Algorithms for Peak-Demand Reduction Maximization with Energy Storage. *ArXiv e-prints* (2021). arXiv:2105.04728.

[25] Yanfang Mo, Qiulin Lin, Minghua Chen, and S. Joe Qin. 2021. Optimal Peak-Minimizing Online Algorithms for Large-Load Users with Energy Storage. In *IEEE INFOCOM*. accepted poster.

[26] John L. Neufeld. 1987. Price discrimination and the adoption of the electricity demand charge. *J. Econ. Hist.* (1987), 693–709.

[27] Guy R. Newsham and Brent G. Bowker. 2010. The effect of utility time-varying pricing and load control strategies on residential summer peak electricity use: A review. *Energy Policy* 38 (2010), 3289–3296.

[28] Junjie Qin, Yinlam Chow, Jiyan Yang, and Ram Rajagopal. 2015. Online modified greedy algorithm for storage control under uncertainty. *IEEE Trans. Power Syst.* 31 (2015), 1729–1743.

[29] Michael J. Risbeck and James B. Rawlings. 2020. Economic model predictive control for time-varying cost and peak demand charge optimization. *IEEE Trans. Autom. Control* 65 (2020), 2957–2968.

[30] Ming Shi, Xiaojun Lin, and Lei Jiao. 2019. On the Value of Look-Ahead in Competitive Online Convex Optimization. *Proc. ACM Meas. Anal. Comput. Syst.* 3, 2, Article 22 (June 2019), 42 pages.

[31] Yuanyuan Shi, Bolun Xu, Di Wang, and Baosen Zhang. 2017. Using battery storage for peak shaving and frequency regulation: Joint optimization for superlinear gains. *IEEE Trans. Power Syst.* 33 (2017), 2882–2894.

[32] D. W. Sobieski and M. P. Bhavaraju. 1985. An economic assessment of battery storage in electric utility systems. *IEEE Trans. Power App. Syst* 12 (1985), 3453–3459.

[33] Wanrong Tang, Suzhi Bi, and Ying Jun Angela Zhang. 2014. Online coordinated charging decision algorithm for electric vehicles without future information. *IEEE Trans. Smart Grid* 5 (2014), 2810–2824.

[34] Moslem Uddin, Mohd Fakhizan Romlie, Mohd Faris Abdullah, Syahirah Abd Halim, Ab Halim Abu Bakar, and Tan Chia Kwang. 2018. A review on peak load shaving strategies. *Renewable and Sustainable Energy Reviews* 82 (2018), 3323–3332.

[35] Rahul Urgaonkar, Bhuvan Urgaonkar, Michael J. Neely, and Anand Sivasubramaniam. 2011. Optimal power cost management using stored energy in data centers. In *Proc. SIGMETRICS*. 221–232.

[36] Rahul Walawalkar, Jay Apt, and Rick Mancini. 2007. Economics of electric energy storage for energy arbitrage and regulation in New York. *Energy Policy* 35 (2007), 2558–2568.

[37] Corey D. White and K. Max Zhang. 2011. Using vehicle-to-grid technology for frequency regulation and peak-load reduction. *J. Power Sources* 196 (2011), 3972–3980.

[38] Xiaomin Xi, Ramteen Sioshansi, and Vincenzo Marano. 2014. A stochastic dynamic programming model for co-optimization of distributed energy storage. *Energy Syst.* 5 (2014), 475–505.

[39] Yue Xiang, Junyong Liu, and Yilu Liu. 2015. Robust energy management of microgrid with uncertain renewable generation and load. *IEEE Trans. Smart Grid* 7 (2015), 1034–1043.

[40] Hong Xu and Baochun Li. 2014. Reducing electricity demand charge for data centers with partial execution. In *Proc. ACM e-Energy*. 51–61.

[41] Hanling Yi, Qiulin Lin, and Minghua Chen. 2019. Balancing cost and dissatisfaction in online EV charging under real-time pricing. In *IEEE INFOCOM*. 1801–1809.

[42] Golbon Zakeri, D. Craigie, Andy Philpott, and M. Todd. 2014. Optimization of demand response through peak shaving. 42 (2014), 97–101.

[43] Ying Zhang, Mohammad H. Hajiesmaili, Sinan Cai, Minghua Chen, and Qi Zhu. 2016. Peak-aware online economic dispatching for microgrids. *IEEE Trans. Smart Grid* 9 (2016), 323–335.

[44] Shizhen Zhao, Xiaojun Lin, and Minghua Chen. 2013. Peak-minimizing online EV charging. In *Ann. Allerton Conf. Commun. Cont. and Comput.* 46–53.

[45] Shizhen Zhao, Xiaojun Lin, and Minghua Chen. 2017. Robust online algorithms for peak-minimizing EV charging under multistage uncertainty. *IEEE Trans. Autom. Control* 62 (2017), 5739–5754.