An Upper Bound on the Convergence Rate of Uplink Power Control in DS-CDMA Systems

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***Abstract*— Uplink transmit power control is crucial for resource allocation and interference management in DS-CDMA systems. In literature, specific distributed algorithms are proposed and their convergence rates are evaluated. In this letter, we explore the optimal achievable convergence rate. We first understand power control as a channel coding problem and derive an upper bound on the convergence rate. Then we propose coding power control bits *across users*, suggesting the achievability of the bound. Finally we use this upper bound to evaluate the uplink power control overhead in IS-95 systems.**

I. Introduction

In Direct Sequence Code-Division Multiple-Access (DS-CDMA) systems, there is no explicitly scheduled time or frequency slot exists among users; hence, uplink power control is crucial for resource allocation and interference management in DS-CDMA systems without multi-user detection, IS-95 for example [1]. In uplink power control, each user varies its access to the resources by adapting its transmit power to the changing channel and interference condition. Resource can be allocated to a new user as long as the Signal to Interference and Noise Ratio (SINR) of every user is satisfied. Hence power control is important for solving the near-far problem in the uplink communication. A power vector is optimal if all users' SINR requirements are satisfied, and at the same time every user's transmit power is at its minimum.

There has been much work in power control since the problem was first introduced by Zander [2] [3]. An excellent framework was provided by Yates [4] and extended in [5], in which the existence of a unique optimal power vector is shown, and the convergence properties of a typical class of iterative algorithms are investigated. Furthermore, the monotonicity property of the power control problem implies it can be solved using distributed algorithms.

Nevertheless, given a feasible optimal power vector exists, one question remains unanswered: what is the optimal convergence rate a power control algorithm can achieve?

In this letter, we explore the answer to this question. It is known that the optimal transmit power vector is a function of the channel gains between users and the base station. Due to the random nature of the channel gains, users do not know this optimal power vector. Power control therefore can be understood as communicating the optimal power vector between the base station and users. Information theory helps to derive a bound on the rate at which a user can eliminate the uncertainty of its optimal transmit power, which directly corresponds to the convergence rate of a power control algorithm.

Achieving the optimal convergence performance in power control requires real-time high rate transmissions for low rate sources. We introduce coding *across users* to meet this requirement in multiuser environments. The base station use a capacity-achieving code to jointly encode the aggregate real-time power control bits of different users. The coded sequence is then sent through a pilot broadcast channel to all users. Each user decodes the entire sequence and extracts its own bits.

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II. Problem Formulation

The model for the uplink power control in a DS-CDMA system with one base station and K users is shown in Fig. (1). At time t, user k adapts its power, denoted by $P_k(t)$, to communicate with the base station. The channel gain between user k and the base station, denoted by $g_k(t)$, is a continuous-time, continuous-value random process. The received power at the base station is $g_k(t)P_k(t)$, denoted by $Q_k(t)$. N_0 is the noise power density; W is the total bandwidth that signals spread on; N_0W is the total noise power at the base station. As the bandwidth W is shared among all K users, other users' signals interfere with user k's signal at the base station.

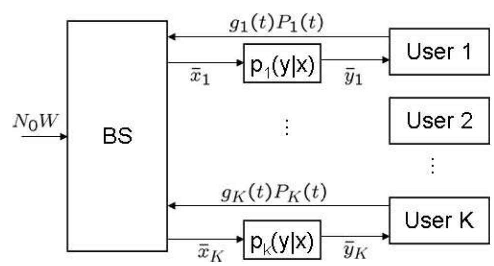

Fig. 1. The uplink power control for a system with one base station and K users.

For simultaneous uplink communications of K users to be feasible, the SINRs of all users have to meet a target. This condition defines a feasible set for users' transmit power, which can be mathematically expressed as follows:

$$\mathcal{P}(t) = \Bigg\{(P_1(t), ...P_K(t)) \,\Bigg|\, \frac{GQ_k(t)}{\sum_{j\neq k} Q_j(t) + N_0W} \geq \beta, \quad P_k(t) \leq \bar{P}_k, Q_k(t) = P_k(t)g_k(t), k = 1, \cdots, K\Bigg\}, \tag{1}$$

where $\bar{P}_k$ is the peak power constraint of user k; G is the processing gain; β is the SINR target.

At a given time t and the optimal point in $\mathcal{P}(t)$, denoted by $P^*(t) = [P_1^*(t), \ldots, P_K^*(t)]^T$, every user is at the minimal possible transmit power so that their SINR requirements are met with equality and no more. Moreover, the corresponding received powers at the base station are the same for all users, denoted by a vector $Q^* = [Q_1^*, \ldots, Q_K^*]^T$:

$$Q_k^* = \frac{N_0 W \beta}{G - \beta(K-1)}, \quad k = 1, \ldots, K. \tag{2}$$

Therefore, $P_k^*(t) = Q_k^*/g_k(t)$ is a function of the random process $\{g_k(t)\}_t$, and is a continuous-time, continuous-value random process.

In the uplink power control used in practice, user k tracks $P_k^*(t)$ with an error tolerance ϵ and a designed frequency f_s, by adapting its transmit power based on the bits $\bar{y}_k$ fed back from base station. That is, user k is satisfied if within time slot m, its transmit power lies in an ϵ-interval around the optimal. f_s and ϵ are design parameters in a DS-CDMA system. Hence, it is equivalent to say user k tracks a discrete-time, discrete-value version of $\{P_k^*(t)\}_t$ by sampling the process at a frequency f_s and quantizing the samples accordingly. We denote the resulting process as $\{P_k^*[m]\}_m$, where m is the time index. Since the power control problem can be distributively solved, it is sufficient to focus on one individual user, e.g. user k, to evaluate the convergence rate of a power control scheme. Hence in the rest of this letter, we will eliminate the user index k, i.e. $P_k^*[m] \to P^*[m]$, unless mentioned otherwise.

In every $1/f_s$ interval, the user tracks $P^*[m]$ using the open-loop and closed-loop power control. The open-loop power control makes a rough estimate of $P^*[m]$, denoted by $P^o[m]$, by inferring from the downlink channel gain strength measured via a pilot signal sent out by the base station. The estimate is typically accurate only up to a few dB, so $P^*[m] \in [P^o[m] - d, P^o[m] + d]$, where d is the open-loop estimation error. Clearly some uncertainty, measured by the entropy $H(P^*[m])$, remains to be eliminated using the closed-loop power control.

The closed-loop power control can be modeled as a communication system shown in Fig. 2. The power control channel, from the base station to the user, is assumed to be memoryless. Therefore the feedback in the system does not affect the channel capacity. Within time slot m, the user tries to eliminate the uncertainty $H(P^*[m])$ by communicating with the base station. Upon receiving each power control symbol y_i, the user eliminates some uncertainty of $P^*[m]$, and adjusts its transmit power, denoted by $P[m]$, accordingly. The base station senses the adapted power $P[m]$, compares it with the optimal $P^*[m]$ [1], and sends out x_{i+1} to start the next power control iteration. In this process, the number of iterations can be counted by the number of symbols received by the user. The convergence rate can hence be defined as *inverse of the number of power control iterations* until the transmit power converges to the optimal value. The less iterations, the higher convergence rate.

[1]In fact the base station's received power is $g[m]P[m]$. It compares this power against the optimal $Q^* = g[m]P^*[m]$. But since $g[m]$ is constant in time slot m, it is equivalent to say the base station senses $P[m]$ and compares it with $P^*[m]$.

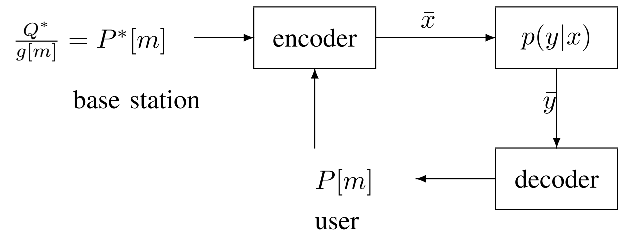

Fig. 2. A discrete communication model for the closed-loop power control in time slot m.

III. The Upper Bound and Coding Across Users

A. The Upper Bound

Let the capacity of the power control channel shown in Fig. (2) be C bits per symbol, the rate of a coding scheme be R bits per symbol. It is well known that $R \leq C$ in the reliable communication. When the user receives N symbols per $1/f_s$ interval, NR bits of uncertainty are eliminated. To eliminate all the uncertainty $H(P_k^*[m])$, the following forms a necessary condition:

$$N\,C \geq N\,R \geq H(P^*[m]). \tag{3}$$

Considering the correlation between $P^*[m]$ and the previous samples $\{P^*[m], i < m\}$, only the conditional entropy $H(P^*[m] \mid P^*[i], i < m)$ needs to be eliminated by the closed-loop power control in time slot m. Hence, in time slot m, the lower bound for N can be explicitly computed as

$$N_{min}[m] = \frac{H\left(P^*[m] \mid P^*[i], i < m\right)}{C}, \quad m = 1, 2, \ldots. \tag{4}$$

The highest convergence rate is the inverse of it; the minimum power control overhead is $N_{min}[m] f_s$ raw bits per second. On the other hand, given $N_{min}[m]$, the maximal tracking frequency in time slot m is $1/(N_{min}[m]T)$ Hz, where T is the time for one closed-loop iteration.

B. Coding Across Users

To achieve the bound computed in the previous subsection, the base station and users need to carry out capacity-achieving reliable transmissions for the power control bits. To achieve the channel capacity, on one hand, a long block code must be applied to encode a long string of information bits. On the other hand, in real-time communication, this requires the source to have a high data rate to generate sufficient bits to encode within a small time interval. However, in power control communication between the base station and users, the source rate of each individual user is low. Therefore, point-to-point real-time capacity-achieving transmissions are impossible in uplink power control.

We introduce coding power control bits *across users* to resolve this conflict, illustrated by Fig. 3. In the scenario of independent coding shown in Fig. 3(a), each user has a low source rate. Hence, it must wait to collect LR bits of information to generate a long coded sequence $x_{k1}, \ldots, x_{kL}, k = 1, \cdots, K$, in order to achieve a high reliable transmission rate R. In power control, this requires the base station to wait to collect the power control bits of one user in different time slots m and code them together.

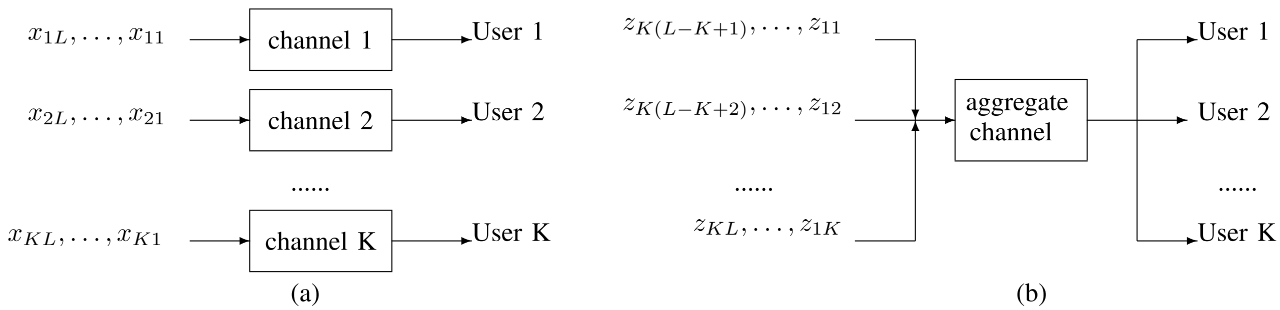

Fig. 3. (a) the scenario of independent coding; (b) the scenario of coding across users (without loss of generality, $L > K$ is assumed).

However, in the scenario of coding *across users* shown in Fig. 3(b), the LR bits of information of different users are jointly encoded to generate a coded sequence $z_{11}, \dots, z_{1L}$ to achieve reliable transmission rate R per user. The sequence are sent through an aggregate broadcast channel. Every user receives the entire coded sequence, and decodes it to get its own bits. When the number of users is K, the time of collecting LR bits of information is reduced K times. The aggregate source rate can be high enough to apply a capacity-achieving code in real-time transmission. In power control, this could correspond to the base station jointly encoding the power control bits of different users within one time slot.

The insight brought by coding *across users* is that the aggregate source rate generated by all users can be high enough to enable real-time capacity-achieving *broadcast* transmissions. The above arguments on independent coding and coding across users hold for arbitrary number of users, as long as the resource, such as bandwidth, is sufficient to support them. The more users, the better. It suggests a way to perform real-time reliable transmissions in a multiuser environment; power control is just one particular application. The disadvantage is that every user needs to decode the entire coded sequence to get its own bits, which seems to be an overkill.

IV. Evaluating IS-95 Power Control Scheme

In this section, we evaluate current IS-95 power control scheme to determine possible room for improvements. The typical settings in IS-95 we used to compute the upper bound are shown in table I [1], [6]. We assume the Rayleigh fading channels between the base station and users are statistically identical; the optimal power process $\{P^*[m]\}_m$ for a user is an i.i.d. random process, and $P^*[m]$ takes discrete values and is uniformly distributed in $\{P^o[m] + j\epsilon; j \in Z, |j\epsilon| \leq d, \text{unit: } dB\}$ in any time slot m.

TABLE I

The typical settings for IS-95 uplink power control

setting	value
f_s: tracking frequency	20Hz
C: achievable rate of downlink channel	0.3 bits per user's symbol
d: open-loop estimation error	4dB
ϵ: closed-loop estimation error requirement	1dB

Under these settings, $H(P^*[m]) = \log_2(d/\epsilon) = 2$ bits for all m. According to (4), the lower bound $N_{min}[m]$ is then 7 for all time slot m, implying power control convergence requires a minimum raw bit rate of 7 bits per $1/f_s$ interval for every user. Hence, the minimum overhead for uplink power control in IS-95 is $7 \times 20 = 140$ raw bits per second, which is much smaller than the 800 raw bits per second overhead in the current IS-95 uplink power control scheme. This implies a possibly huge room for improvement. From another point of view, with 800 raw bits per second overhead, users are able to track the optimal transmit powers at about 100Hz – tight power control at such high frequency could even be used to mitigate fast fades.

V. Discussion and Conclusion

In this letter, we provide an upper bound on the convergence rate of uplink power control in DS-CDMA systems, where convergence rate is defined as inverse of the number of power control iterations until the transmit power converges to the optimal value. The analysis applies to any system that needs tight power control. We propose coding power control bits *across users* to perform the real-time high rate reliable transmissions, suggesting the bound is possible to achieve. This idea suggests a general way to carry out low source rate, real-time, capacity-achieving, broadcast transmission in multiuser environments.

An open issue is to design an algorithm achieving the best convergence performance. Whether the SINR requirement can be kept satisfied during the convergence process is also unclear. The answers of these questions would encourage the practical implementation of the optimal power control in DS-CDMA systems.

Acknowledgement

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