Stay or *Switch*: Competitive Online Algorithms for Energy Plan Selection in Energy Markets with Retail Choice

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ABSTRACT

Energy markets with retail choice enable customers to switch energy plans among competitive retail suppliers. Despite the promising benefits of more affordable prices and better savings to customers, there appears subsided participation in energy retail markets from residential customers. One major reason is the complex online decision-making process for selecting the best energy plan from a multitude of options that hinders average consumers. In this paper, we shed light on the online energy plan selection problem by providing effective competitive online algorithms. We first formulate the online energy plan selection problem as a metrical task system problem with temporally dependent switching costs. For the case of constant cancellation fee, we present a 3-competitive deterministic online algorithm and a 2-competitive randomized online algorithm for solving the energy plan selection problem. We show that the two competitive ratios are the best possible among deterministic and randomized online algorithms, respectively. We further extend our online algorithms to the case where the cancellation fee is linearly proportional to the residual contract duration. Through empirical evaluations using real-world household and energy plan data, we show that our deterministic online algorithm can produce on average 14.6% cost saving, as compared to 16.2% by the offline optimal algorithm, while our randomized online algorithm can further improve cost saving by up to 0.5%.

CCS CONCEPTS

• **Theory of computation → Online algorithms**.

KEYWORDS

retail choice, energy markets, energy plans, competitive analysis

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1 INTRODUCTION

Retail choice in energy markets aims to provide diverse options to residential, industrial and commercial customers by enabling selectable purchases of electricity and natural gas from multiple competitive retail suppliers [26]. Traditionally, the energy sector is a notoriously monopolized industry with vertically integrated providers spanning energy generation, transmission and distribution. Limited options of energy suppliers and tariff schemes have been available in most of the world. However, the energy sector is being restructured by deregulation, and new legislation has been launched worldwide to promote more competitive energy retail markets, giving customers higher transparency and more options. Liberating the energy retail markets to competitive suppliers not only allows more affordable tariff schemes and better savings to customers, but it also encourages more customer-oriented and flexible services from suppliers. With more bargaining leverage, customers can also influence the energy suppliers to be more socially conscious and sustainable toward a low-carbon society. Furthermore, the emergence of virtual power plants [24], where household PVs and batteries are aggregated as an alternate supplier, can become a new form of energy suppliers in competitive retail markets.

In competitive energy retail markets (including electricity and natural gas), there is a separation between utility providers (who are responsible for the management of energy transmission and distribution infrastructure, as well as the maintenance for ensuring its reliability) and energy suppliers (who generate and deliver energy to utility providers). Energy suppliers are supposed to compete in an open marketplace by providing a range of services and tariff schemes. With retail choice, customers can shop around and compare different energy plans from multiple suppliers, and then determine the best energy plans that suit their needs. The switching from one supplier to another can be attained conveniently via an online platform, or through third-party assistance services.

Since the deregulation in the energy sector in several countries, there has been a blossom of energy retail markets. As of 2018, there are 23 countries in the world offering energy retail choice, including the US, the UK, Australia, New Zealand, Denmark, Finland, Germany, Italy, the Netherlands and Norway [18]. In the US, there are over 13 states offering electricity retail choice [16, 26]. In particular, there are over 109 retail electric providers in Texas offering more than 440 energy plans, including 97 of which generated from all renewable energy sources [10]. In the UK, there are over 73 electricity and natural gas suppliers [20]. In Australia, there are over 33 electricity and natural gas retailers and brands [2].

Despite the promising goals of retail choice in energy markets, there however appears subsided participation particularly from residential customers. Declining residential participation rates and

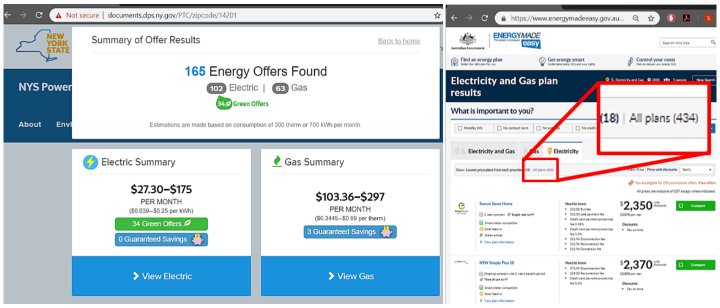

Figure 1: Examples showing a large number of energy plans in practice. There are 165 energy plans available in Buffalo NY (zip code: 14201) and 434 electricity plans available in Sydney (postal code: 2000). (Sources: http://documents.dps.ny.gov/PTC; http://EnergyMadeEasy.gov.au).

diminishing market shares of competitive retailers have been reported in the recent years [1, 12]. While there are several reasons behind the subpar customer reactions, one identified major reason is the confusion and complication of the available energy plans in energy retail markets [2]. First, there are rather complex and confusing tariff structures by energy retailers. It is not straightforward for average consumers to comprehend the details of consumption charges and different tariff schemes. Second, savings and incentives among energy retailers are not easy to compare. Some discounts are conditional on ambiguous contract terms. Without discerning the expected benefits of switching their energy plans, most customers are reluctant to participate in energy retail markets. Third, there lack proper evaluation tools for customers to keep track of their energy usage and expenditure. Last, the increasing market complexity with a growing number of retailers and agents obscures the benefits of retail choice. For example, using real-world official datasets, we found 165 energy plans available in Buffalo NY (zip code: 14201), and 434 electricity plans available in Sydney (postal code: 2000). See the screenshots in Fig 1. A large number of available energy plans cause considerable confusion and complexity to average customers.

To bolster customers' participation, several government authorities and regulators have launched websites and programs to educate customers the benefits of retail choice in energy markets [19, 21–23]. Recently, a number of start-up companies emerged to capitalize the opportunities of retail choice in energy markets by providing assistance services and online tools to automatically determine the best energy plans for customers as well as offering personalized selection advice. These assistance services and online tools are integral to the success of retail choice in energy markets by automating the confusing and complex decision-making processes of energy plan selections. In the future, household PVs, batteries, and smart appliance will optimize their usage and performance in conjunction with energy plan selection. Therefore, we anticipate the importance of proper decision-making processes for energy plan selection in energy markets with retail choice.

There are several challenging research questions arisen in the decision-making processes for energy plan selection:

(1) **Complex Tariff Structures:** There are diverse tariff structures and properties in practical energy plans. For example, the tariff schemes may have different contract periods (e.g., 6 months, one year, or two years). There may be different time-of-use and peak tariffs as well as dynamic pricing depending on renewable energy sources. Also, it is common to have various administrative fees, such as connection, disconnection, setup and cancellation fees. When there are rooftop PVs or home batteries, there will be feed-in tariffs for injecting electricity to the grid. In some countries, different appliances can be charged by different rates (e.g., boilers and heaters are charged differently).

(2) **Uncertain Future Information:** To decide the best energy plan that may last for a long period, one has to estimate the future usage and fluctuation of energy tariffs. It is difficult to predict the uncertain information accurately to make the best decisions. For example, energy tariffs may depend on global energy markets. Unexpected guests or travel plans will largely effect energy demand of consumers. If there are rooftop PVs, their performance is conditional on the unpredictable weather. The various sources of uncertainty complicate the decision-making process of energy plan selection.

(3) **Assurance of Online Decision-Making:** The energy plan selection decision-making process is determined over time when new information is gradually revealed (e.g., the present demand and updated tariffs). A sequential decision-making process with a sequence of gradually revealed events is called *online decision-making*. The average customers are reluctant to switch to a new energy plan, unless there is certain assurance provided to their selected decisions. The online decision-making process should incorporate suitable bounds on the optimality of decisions as a metric of confidence for customers.

In this paper, we shed light on the online energy plan selection problem with practical tariff structures by offering effective algorithms. Our algorithms will enable automatic systems that monitor customers' energy consumption and newly available energy plans from an energy market with retail choice, and automatically recommend customers the best plans to maximize their savings. We note that a similar idea has been explored recently in practice [13].

Our results are based on *competitive online algorithms*. Decision-making processes with incomplete knowledge of future events are common in daily life. For decades, computer scientists and operations researchers have been studying sequential decision-making processes with a sequence of gradually revealed events, and analyzing the best possible strategies regarding the incomplete knowledge of future events. For example, see [8] and the references therein. Such strategies are commonly known as *online algorithms*. The analysis of online algorithms considering the worst-case impacts of incomplete knowledge of future events is called *competitive analysis*. Competitive online algorithms can provide assurance to online decision-making processes with uncertain future information.

We summarize our contributions in the following.

- ▷ We present a case study in Section 2 to evaluate the potential saving of selecting an appropriate energy plan using real-world data of household demands and energy plan prices.

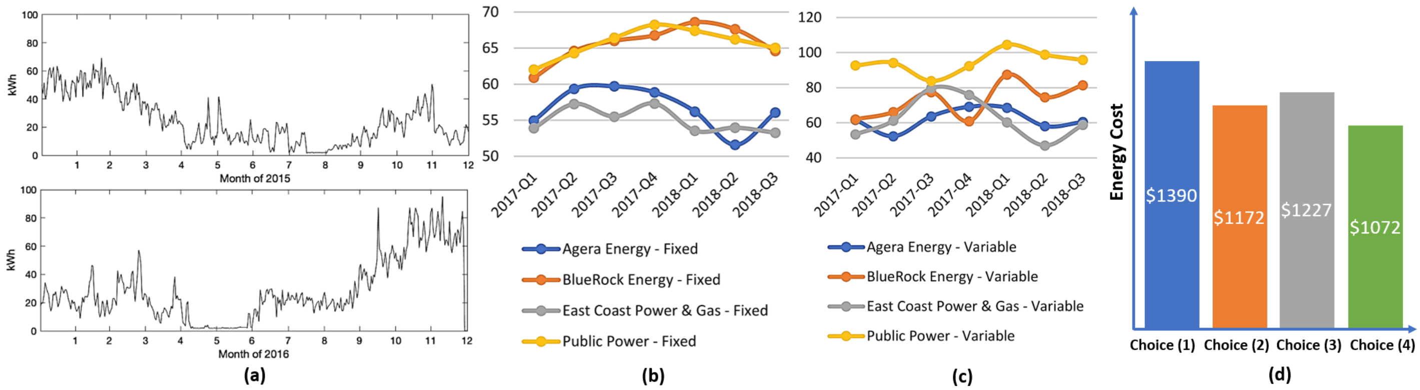

Figure 2: (a) Electricity consumption profile of a particular family in 2015-2016, plotted in kWh against the month of year. (Source: Smart* project [7]). (b)-(c) Weighted average historic energy plan prices from four electricity suppliers in 10001, NY, US. ($ = rate * est. avg. mth usage (700kWh)) (b) shows 12-month fixed-rate energy plans, whereas (c) shows one-month variable-rate energy plans. (Source: http://documents.dps.ny.gov/PTC/). (d) Energy costs by Choices (1)-(4) in Section 2.2.1.

- ▷ We formulate the online energy plan selection problem as a metrical task system problem with temporally dependent switching cost in Section 3.
- ▷ For constant cancellation fee, the problem is reduced to a discrete online convex optimization problem. In Section 4, we present an offline optimal algorithm, a 3-competitive deterministic online algorithm, and a 2-competitive randomized online algorithm for solving the energy plan selection problem. These algorithms maintain great consistency while achieving their optimality.
- ▷ We extend our study to consider cancellation fee to be temporally dependent on the residual contract period by incorporating additional constraints in Section 5. We conjecture that our proposed algorithms should preserve their competitiveness under proper modification.
- ▷ Through empirical evaluations using real-world data in Section 6, we show that our deterministic online algorithm can produce on average 14.6% cost saving, as compared to 16.2% by the offline optimal algorithm. Our randomized online algorithm can further improve the cost saving.

Due to space limit, all proofs are presented in the technical report [25], and we refer interested readers to it for details.

2 CASE STUDY

To motivate our results, we first present a case study to evaluate the potential saving of selecting an appropriate energy plan. We use real-world data of household demands and retailers' energy plan prices to estimate the costs and savings of selecting different energy plans.

2.1 Dataset

2.1.1 Household Electricity Consumption. We use the electricity consumption data from Smart* project [7], which consists of 114 single-family apartments recorded in 2015-2016. A typical family's consumption profile in two years is plotted in Figure 2 (a). By comparing the consumption pattern in two consecutive years, we observe dramatic fluctuations from 2015 to 2016. Hence, the consumption rate of the previous year may not be a reliable indicator for selecting the next year's energy plan.

2.1.2 Energy Plan Prices. We consider the historic data of energy plan prices from the real-world official dataset in New York State. From Figure 2 (b)-(c), we observe that variable-rate energy plans have a large price range, whereas fixed-rate energy plans have a stable price range. Note that fixed-rate energy plans are associated with different kinds of fees when customers switch their selections. For example, cancellation fee applies in the US when cancelling a long-term plan, whereas connection/disconnection fee applies in Australia when switching to other retailers. At the same time, fixed-rate energy plans have additional fee when the usage differs greatly from previous year.

2.2 Potential Savings

2.2.1 Setting. We consider the case of a household switching between a fixed-rate energy plan and a variable-rate energy plan in one year. Our evaluation is based on the consumption data and energy plan prices in Figure 2 (a)-(c). Suppose that a household having the same consumption as is shown in 2016, with previous years data as in 2015, and selects Public Power as the retailer, which provides either a variable-rate energy plan or a 12-month fixed-rate energy plan with the rate in 2017. We consider four representative choices as follows:

(1) remaining in a variable-rate plan,
(2) remaining in a fixed-rate plan,
(3) first staying in a fixed-rate plan, and then cancelling with $100 cancellation fee to switch to a variable-rate plan, or
(4) first staying in a variable-rate plan, and then switching to a fixed-rate plan without switching cost.

2.2.2 Observations. The energy costs under different choices are plotted in Figure 2 (d). Since the household consumption varies considerably from 2015 to 2016, remaining in a variable-rate plan (Choice (1)) will cost more than other energy plans. On the other hand, remaining in a fixed-rate plan (Choice (2)) can save 15.68% cost. However, staying in a fixed-rate plan in the beginning and cancelling before the end of contract (Choice (3)) only saves 11.73% cost due to the cancellation fee. The best choice is Choice (4) - staying in a variable-rate plan at first and then switching to a fixed-rate plan, which saves up to 22.88% cost. The reason is that the consumption was much lower in January and February 2016 than 2015. Therefore, it is not economical to sign up a fixed-rate plan too early. The high consumption in September and onwards makes a fixed-rate plan more economical.

Note that we assume a single retailer without switching to another retailer. It can be shown later that an even bigger saving can be achieved by switching to another retailer.

In this section, we showed the substantial potential savings by selecting a proper energy plan. In practice, there may be many more retailers with a large number of energy plans. Hence, it requires a systematic solution for the energy plan selection problem.

3 PROBLEM FORMULATION

To solve the energy plan selection problem, we first present a formal model of it. We consider a typical setting where a consumer can select his/her energy plan offered by various energy retailers. We formulate the energy plan selection problem in **EPSP**, which considers arbitrary demands, time-varying prices, and cancellation fees. Since energy plans differ considerably from one country to another, we consider a typical energy plan model in the US. Some key notations are defined in Table 1 and acronyms are listed in Table 2.

Table 1: Key notations.

Notation	Definition
T	The total number of time intervals (unit: month)
β_t	The cancellation fee at time t ($)
σ_t	The joint input at time t
s_t	The selected plan at time t (variable-rate plan denoted by '1' and fixed-rate plan denoted by '0')
e_t	The electricity demand of customer at time t (kWh)
p_t^0	The price per unit of electricity usage for fixed-rate plan ($ / kWh)
p_t^1	The price per unit of electricity usage for variable-rate plan ($ / kWh)
B_t	The base load for fixed-rate plan (kWh)
H	The underusage charging rate for fixed-rate plan ($ / kWh)
R	The minimum contract length for fixed-rate plan (month)
L	The total length of contract for fixed-rate plan (month)
α	The cancellation fee for the residual time in the contract ($ / month)

Table 2: Acronyms for problems and algorithms.

Acronym	Meaning
EPSP	Energy Plan Selection Problem
SP	Simplified version of **EPSP**
dSP	**EPSP** with linearly decreasing switching costs
OFA_s	The optimal offline algorithm for **SP**
gCHASE_s	Generalized version of deterministic online algorithm CHASE_s for **SP**
gCHASE_s^r	Randomized version of gCHASE_s for **SP**

3.1 Model

3.1.1 Uncertain Electricity Demands. We consider arbitrary consumers demand throughout the whole period. Let the electricity demand at time t be $e(t)$. Note that we do not rely on historic data for any prediction, nor any stochastic model of $e(t)$.

3.1.2 Pricing Schemes. We describe the pricing schemes of different energy plans in the US. We first consider two major energy plans to be selected by a customer. Let s_t be the selected plan, where 0 represents a fixed-rate plan and 1 represents a variable-rate plan.

- *Variable-Rate Plan*: Switching between different variable-rate plans will not incur any cost. As a result, we can always assume the least-cost variable-rate plan when selecting a variable-rate plan. Hence, there is only a single variable-rate plan to be considered at each time. Denote by p_t^1 the price per electricity unit for the variable-rate plan, and the consumption charge at time t is $p_t^1 \cdot e_t$.
- *Fixed-Rate Plan*: Fixed-rate plans also vary with market prices, but do not fluctuate as much as variable-rate plans and are characterized by a stable ranking for a long period of time. Moreover, there is always a relatively high cancellation fee when cancelling a fixed-rate plan. With this understanding, we assume that a consumer will not switch between fixed-rate plans.
  Note that a fixed-rate plan will not offer the same electricity price for arbitrary demand, but a tiered-pricing scheme up for a certain level of demand [11]. Let B_t be a base load for a consumer for each time t in the previous year, and p_t^0 be the price per electricity unit for a fixed-rate plan. A consumer will pay $B_t \cdot p_t^0$ if his/her demand is between $0.9B_t$ and $1.1B_t$. Otherwise, an underusage fee will be charged at rate H when e_t is less than $0.9B_t$, or an overusage fee will be charged at the same rate of a variable-rate plan when e_t is more than $1.1B_t$.

We define the cost function $g_t(s_t)$ based on the input $\sigma_t \triangleq (e_t, p_t^0, p_t^1, B_t)$ and the selected plan s_t at time t as:

$$g_t(s_t) \triangleq \begin{cases} e_t \cdot p_t^1, & \text{if } s_t = 1; \\ e_t \cdot p_t^0 + (p_t^1 - p_t^0) \cdot (e_t - 1.1B_t)^+ & \text{if } s_t = 0. \\ \quad - H \cdot (0.9B_t - e_t)^+, & \end{cases} \quad (1)$$

The model can be extended to describe other advanced settings, including the one in Australia [23]. We only focus on the one in (1) in this paper.

3.1.3 Cancellation Fee. When a customer cancels a fixed-rate plan, there will be a cancellation fee of the following types:

(1) *No Fee*: The customer does not need to pay any cancellation fee but s/he needs to inform the retailer in advance (e.g., 30 days ahead).
(2) *Constant Fee*: If the residual number of months in the contract is within a certain level, then a fixed amount of cancel fee is required (e.g., $100 if less than 12 months left in the contract, or $200 if between 12 to 24 months left in the contract).
(3) *Temporally Linear-dependent Fee*: a pre-specified charge times the residual number of months in the contract (e.g., $10 per remaining month in the contract).

Let β_t be the cancellation fee at time t. We note that within each period of a fixed-rate plan, it is either a constant, or a linearly decreasing value proportional to the residual contract period.

3.1.4 Contract Period. The retailers often offer several contracts with various contract periods and different restrictions. Since variable-rate plans have higher per-unit rates than those of fixed-rate plans, it is uncommon that variable-rate plans are limited by a contract period. However, retailers may want to retain consumers in a fixed-rate plan by stipulating a certain fixed period of committing to the contract. Denote by R the minimum contract length for a fixed-rate plan, by which a consumer shall not switch to a variable-rate plan during this period. Let L be the total contract length, so that consumers will not need to pay any cancellation fee when terminate by the end of contract.

3.2 Problem Definition

The total time period T is divided into integer slots $\mathcal{T} \triangleq \{1, \cdots, T\}$, each is assumed to last for one month, corresponding to the available minimum contract period. Our goal is to find a solution $\boldsymbol{s} \triangleq (s_1, s_2, \cdots, s_T)$ to the following energy plan selection problem:

$$\textbf{(EPSP)} \quad \min \ \text{Cost}(\boldsymbol{s}) \triangleq \sum_{t=1}^{T} \left(g_t(s_t) + \hat{\beta}_t \cdot (s_t - s_{t-1})^+ \right) \tag{2a}$$

$$\text{subject to} \quad s_\tau \le \mathbf{1}_{\{s_t \ge s_{(t-1)}\}}, t+1 \le \tau \le t + \mathsf{R} - 1, \tag{2b}$$

$$\hat{\beta}_t = \mathbf{1}_{\{\sum_{t-\mathsf{L}+1}^{t} s_\tau > 0\}} \cdot \beta_t, \tag{2c}$$

$$\text{variables} \quad s_t \in \mathcal{N} \triangleq \{0, 1\}, t \in [1, T],$$

where $(x)^+ = \max\{x, 0\}$ and $\mathbf{1}_{\{\cdot\}}$ is an indicator function. Function $g_t(s_t)$ is defined in (1). Without loss of generality, the initial state s_0 is set to be 0. The total cost function in (2a) is consisted of operational cost $g_t(s_t)$ and switching cost when cancelling a fixed-rate plan. The constraints in (2b) capture the minimum contract length for the fixed-rate plan. The constraints in (2c) capture zero cancellation fee when terminating a fixed-rate plan exactly by the end of contract.

Remarks: Our problem formulation bears similarity with the online optimization problems in LCP [15], CHASE [17] or their extended version [3, 6, 14]. However, there is a fundamental difference that makes our problem more challenging. In the constraints (2c), we note that the cancellation fee depends on the last L states, which cannot be reduced to a sub-problem as in the prior studies. Thus, our problem is harder and requires non-trivial treatments.

4 CONSTANT SWITCHING COST

In this section, we consider a simplified version of this problem by restricting our attention to the essential part. First, by a survey of the existing energy plans in [21], we observe that it is common for retailers to offer fixed-rate plans without minimum contract period. Further, if there is a contract period, the longer the contract period, the lower price it has. Under such setting, we can drop the constraints in (2b) and (2c) by assuming R is 0 and L approaches infinity with respect to T. Moreover, we focus on the setting with constant cancellation fee, which is also common in many fixed-rate plans.

This basic problem can be reformulated as follows:

$$\textbf{(SP)} \qquad \min \ \text{Cost}(\boldsymbol{s}) \triangleq \sum_{t=1}^{T} \left(g_t(s_t) + \beta \cdot (s_t - s_{t-1})^+ \right) \tag{3a}$$

$$\text{variables} \quad s_t \in \mathcal{N}, \tag{3b}$$

Note that this problem involves only two states as the decision variables, and the switching cost β depends on the current consecutive time slots. Proposition 1 shows that problem **SP** (or equivalently, **P1**) is equivalent to **P2**, which is a classical online decision problem known as the *Metrical Task System* problem [8].

Proposition 1. *The following two problems are equivalent under the boundary condition of* $s_0 = s_{T+1} = 0$, $g_{T+1}(0) = 0$*:*

$$\textbf{(P1)} \qquad \min \sum_{t=1}^{T+1} \left(g_t(s_t) + \beta (s_t - s_{t-1})^+ \right) \tag{4}$$

$$\text{subject to} \quad s_t \in \mathcal{S} \triangleq \{0, 1, \cdots, n\}$$

$$\textbf{(P2)} \qquad \min \sum_{t=1}^{T+1} \left(g_t(s_t) + \frac{\beta}{2} \left| s_t - s_{t-1} \right| \right) \tag{5}$$

$$\text{subject to} \quad s_t \in \mathcal{S} \triangleq \{0, 1, \cdots, n\}$$

4.1 Offline Optimal Algorithm

In this section, we provide an offline optimal solution to the problem **SP**, where input $\boldsymbol{\sigma}$ is given in advance. Our offline optimal solution (OFA_s) does not incur high space and computational complexity, and hence can be implemented efficiently. The offline optimal solution will motivate our design of proper online algorithm in the next section. The basic idea is based on the theoretical framework in [17], from which we adopt similar notations.

First, we note that the cost function $g_t(\cdot)$ is non-negative. Hence, we can focus on the cost difference between two states.

Definition 1. *Define the one-timeslot cost difference by*

$$\delta(t) \triangleq g_t(0) - g_t(1). \tag{6}$$

Positive $\delta(t)$ suggests changing to state 1, or otherwise, state 0. Next, we define the cost difference for consequent time slots.

Definition 2. *Define the cumulative cost difference by*

$$\Delta(t) \triangleq \left(\Delta(t-1) + \delta(t) \right)_{-\beta}^{0}, \tag{7}$$

where $(x)_a^b \triangleq \min\{b, \max\{x, a\}\}$*. The initial condition is* $\Delta(0) = -\beta$*.*

Algorithm 1 OFA_s

```
Set s_{T+1} ← 0
for t from T to 1 do
    Compute Δ(t)
    if Δ(t) = −β then
        s_t ← 0
    else if Δ(t) = 0 then
        s_t ← 1
    else
        s_t ← s_{t+1}
    end if
    return s_t
end for
```

Algorithm 2 gCHASE_s

```
Set s_0 ← 0
for t from 1 to T do
    Compute Δ(t)
    if Δ(t) = −β then
        s_t ← 0
    else if Δ(t) = 0 then
        s_t ← 1
    else
        s_t ← s_{t−1}
    end if
    return s_t
end for
```

If $\Delta(t)$ increases to 0, it means that staying at state 0 will cost more than changing to state 1, and hence, it is more desirable to change to state 1 afterwards. Otherwise, it is more desirable to change to state 0 if $\Delta(t)$ decreases to $-\beta$.

Theorem 1. *OFA_s (Algorithm 1) is an offline optimal algorithm for problem **SP**.*

Proof. (Sketch) At first, we need to show the behavior of OFA_s is the same as the optimal solution in [17], which is evident followed by its definition. Meanwhile, we note that OFA_s is also similar to the one in [15], in which the property of the solution vector is related by a backward recurrence relation. □

Theorem 2. *Both the running time and space requirement of OFA_s for problem **SP** are $O(T)$.*

Furthermore, we claim that OFA_s is still an offline optimal algorithm when taking time and space complexity into consideration. It is evident that no algorithm can run faster than $O(T)$ because of the length of the input sequence. Meanwhile, to store the solution vector, a minimum space of $O(T)$ is necessary.

4.2 Competitive Online Algorithm

This section is divided into two parts. First, a deterministic online algorithm is presented to output a deterministic solution over time. Second, a randomized online algorithm is presented to generate a probabilistic ensemble of solutions over time.

4.2.1 Deterministic Online Algorithm.

We formally define online algorithm and competitive ratio [8] first. Let the input to the problem be $\boldsymbol{\sigma} = (\sigma_t)_{t=1}^{T}$. Given $\boldsymbol{\sigma}$ in advance, the problem can be solved offline optimally. Let $\text{Opt}(\boldsymbol{\sigma})$ be the offline optimal cost for input $\boldsymbol{\sigma}$, which is given in advance. A deterministic *online algorithm* $\mathcal{A}$ decides each output s_t deterministically only based on $(\sigma_\tau)_{\tau=1}^{t}$. We say the online algorithm $\mathcal{A}$ is *c-competitive* if

$$\text{Cost}_{\mathcal{A}}(s_0, \boldsymbol{\sigma}) \leq c \cdot \text{Opt}(s_0, \boldsymbol{\sigma}) + \gamma(s_0), \forall \boldsymbol{\sigma}, \tag{8}$$

where s_0 is the initial state at time 0 defined by the problem formulation, and $\gamma(s_0)$ is a constant value only depended on s_0. The smaller c is, the better online algorithm $\mathcal{A}$ is. The smallest c satisfying (8) is also named as the *competitive ratio* of $\mathcal{A}$. We will devise a competitive online algorithm for problem **SP**.

From Algorithm 1, we observe that the optimal solution will certainly stay in state 1 when $\Delta(t)$ is 0, and stay in state 0 when $\Delta(t)$ is $-\beta$. Hence, we replicate such behavior in an online fashion as an online algorithm in gCHASE_s (Algorithm 2), which appears to "chase" OFA_s in a literal sense.

Theorem 3. *The competitive ratio of gCHASE_s (Algorithm 2) for problem **SP** is 3.*

Proof. (Sketch) We classify the time intervals into several critical segments, as in [17]. Then we compare the costs for each type of these segments between gCHASE_s and OFA_s. By combining all segments together and upper bounding their cost ratio, we can obtain the overall competitive ratio as 3. □

Theorem 4. *The lower bound on competitive ratio of any deterministic online algorithms for problem **SP** is 3.*

Proof. (Sketch) We adopt the similar ideas from [3, 17] and [6]. A specific input sequence is constructed progressively depending on the behavior of a given online algorithm $\mathcal{A}$. Then, we bound the cost of $\mathcal{A}$ and the minimum cost for an offline optimal algorithm by 3. □

4.2.2 Randomized Online Algorithm.

If an online algorithm $\mathcal{A}$ is a randomized algorithm (namely, making decisions probabilistically), we define the *expected* competitive ratio of $\mathcal{A}$ by the smallest constant c satisfying

$$\mathbb{E}[\text{Cost}_{\mathcal{A}}(s_0, \boldsymbol{\sigma})] \leq c \cdot \text{Opt}(s_0, \boldsymbol{\sigma}) + \gamma(s_0), \forall \boldsymbol{\sigma}, \tag{9}$$

where $\mathbb{E}[\cdot]$ is the expectation over all random decisions. We next devise a competitive randomized online algorithm for problem **SP**.

Instead of changing the state only at the moments of observing the cumulative cost difference $\Delta(t)$ reaching 0 or $-\beta$, we introduce randomization to change the state at an earlier random moment. The basic idea is that for increasing $\Delta(t)$, the faster it increases, the higher probability of changing the state to 1. On the other hand, for decreasing $\Delta(t)$, the faster it decreases, the higher probability of changing the state to 0. This idea was first proposed in [3] and [6].

We remark that randomized algorithms do not necessarily entail random decisions of a single customer. When we consider an ensemble of a large number of customers using an automatic energy plan recommendation system, each customer can be given a deterministic decision rule drawn from a probabilistic ensemble of

Algorithm 3 gCHASE$_s^r$

```
Set s_0 ← 0
for t from 1 to T do
  Compute Δ(t)
  if Δ(t) = 0 then
    s_t ← 1
  else if Δ(t) = −β then
    s_t ← 0
  else
    if Δ(t − 1) ≤ Δ(t) then
      if s_{t−1} = 1 then
        s_t ← 1
      else
        s_t ← 0 with probability Δ(t)/Δ(t−1)
        s_t ← 1 with probability 1 − Δ(t)/Δ(t−1)
      end if
    end if
  else
    if s_{t−1} = 0 then
      s_t ← 0
    else
      s_t ← 0 with probability 1 − (β+Δ(t))/(β+Δ(t−1))
      s_t ← 1 with probability (β+Δ(t))/(β+Δ(t−1))
    end if
  end if
  return s_t
end for
```

decision rules. In the end, the expected cost of a customer can be computed by the expected cost of a randomized algorithm.

Theorem 5. *The expected competitive ratio of gCHASE$_s^r$ (Algorithm 3) for problem **SP** is 2.*

Proof. (Sketch) By relaxing the discrete states to a continuous setting, we show that the expected cost of gCHASE$_s^r$ is equal to a continuous version of gCHASE$_s^r$. Then we show that the continuous version of gCHASE$_s^r$ has a competitive ratio of 2 in the continuous setting. Lastly, it can be verified that the optimal cost in the discrete setting is an upper bound to the continuous one. □

Theorem 6. *The lower bound on expected competitive ratio of any randomized online algorithm for problem **SP** is 2.*

Proof. (Sketch) The proof is similar to the ones in [4] and [5]. First, the expected cost of an arbitrary algorithm $\mathcal{A}$ is not smaller than a converted deterministic algorithm $\mathcal{A}^*$ in the continuous setting. Then a 2-competitive deterministic algorithm $\mathcal{B}$ is constructed in the continuous setting to provide the lower bound on the cost of any $\mathcal{A}^*$. Lastly, use the discrete setting to provide an upper bound on the cost of an offline optimal algorithm. □

5 LINEARLY DECREASING CANCELLATION FEE

In this section, we consider the extension to the setting with linearly decreasing cancellation fee by adding back constraint (2c) in **EPSP**. Note that in our study, β_t is proportional to the amount of residual time remaining in a fixed-rate plan, we consider that the original problem should be properly reformulated in order to taking it into account. We propose another problem setting in the following, and discuss its inherited connection with problem **SP**.

First, we divide the total time period $[0, T+1]$ into consecutive time segments, such that states remain unchanged within the same segment:

$$[T_0, T_1 - 1], \cdots, [T_n, T_{n+1} - 1], \cdots, [T_{2n}, T_{2n+1} - 1], \tag{10}$$

where $T_0 = 0$, $T_{2n+1} - 1 = T + 1$, and

$$s_t = \begin{cases} 0, & t \in [T_{2i}, T_{2i+1} - 1], \quad \forall i \in [0, n], & (11a) \\ 1, & t \in [T_{2i-1}, T_{2i} - 1], \quad \forall i \in [1, n]. & (11b) \end{cases}$$

We assume that a consumer is not allowed to maintain a fixed-rate plan longer than its total length L. This is reasonable because after the contract is expired, the consumer will be automatically switched to a variable-rate plan, if s\he has not explicitly renewed the contract. Based on the above assumption, we rewrite the problem objective and constraints in below.

$$\textbf{dSP:} \min \ \text{Cost}(s) \triangleq \sum_{t=1}^{T} g_t(s_t) + \sum_{i=0}^{n} \Big(\alpha \cdot [\mathsf{L} - (T_{2i+1} - T_{2i})]\Big) \tag{12a}$$

$$\text{subject to} \ \ T_{2i+1} - T_{2i} \le \mathsf{L}, \forall i \in [0, n], \tag{12b}$$

$$\text{variables} \ \ s_t \in \mathcal{N}, 2n \in [0, T], \tag{12c}$$

where $\alpha \cdot [\mathsf{L} - (T_{2i+1} - T_{2i})]$ is the cancellation fee each time for cancelling a fixed-rate plan. Constraint (12b) captures the fact that staying in a fixed-rate plan cannot exceed its maximum length L.

Similar to problem **SP**, we use Definition 1 for $\delta(t)$, but the cumulative cost difference $\Delta(t)$ is replaced by $\hat{\Delta}(t)$ as follows.

Definition 3. *Define the cumulative cost difference for **dSP** by*

$$\hat{\Delta}(t) \triangleq \Big(\hat{\Delta}(t-1) + \delta(t) - \alpha\Big)_{-\beta}^{0}, \tag{13}$$

where $\beta = \alpha \cdot \mathsf{L}$. Initially set $\hat{\Delta}(0) = -\beta$.

Intuitively, we can divide the total cancellation fee $\alpha \cdot \mathsf{L}$ into L time slots. Then in each time slot, the fixed-rate plan suffers α more than the variable-rate plan due to its potential cancellation fee. Because of the similar structure between **dSP** and **SP**, we can now use gCHASE$_s$ as a heuristic online algorithm by replacing $\Delta(t)$ with $\hat{\Delta}(t)$ to solve **dSP**. In Section 6, we show by empirical evaluations that such a heuristic online algorithm can indeed produce a good approximation to an offline optimal solution. We further conjecture that the competitive ratio when applying heuristic online algorithm gCHASE$_s$ to **dSP** is $(3 + \frac{1}{\mathsf{L}-1})$. Namely, the longer the contract period L is, the more competitive the heuristic online algorithm is.

Note that OFA$_s$ is not an optimal offline algorithm for **dSP**, because backward recurrence relation is not in compliance with the linearly decreasing switching cost scenario. Borrowing ideas on constructing offline algorithms from [3, 17], it is always possible to use dynamic programming to solve this problem or similar ones. Even though dynamic programming may lead to high time and space complexity, it is so far the best one in our mind which can guarantee the optimal results. Hence, we will use dynamic programming to obtain the empirical optimal solutions instead of OFA$_s$ in

Section 6. Furthermore, we anticipate that the randomized online algorithm gCHASE_s^r should also be competitive after changing to the new $\hat{\Delta}(t)$.

6 EMPIRICAL EVALUATIONS

In this section, we evaluate our proposed algorithms under different settings using real-world traces. Our objectives are threefold: (i) estimating the potential savings by switching energy plans as compared to staying in a variable-rate plan, (ii) comparing the performances of both deterministic and randomized online algorithms against the offline optimal algorithm, and (iii) analyzing the effect of changing the cancellation fee.

6.1 Dataset and Parameters

6.1.1 Electricity Demand. The demand traces are from Smart* project [7]. We use the electrical dataset which involves 114 single-family apartments for the period 2015-2016. Their monthly average consumption is around 765 kWh, which matches with the monthly average consumption in the US [22]. As shown in Figure 2(a), the daily consumption pattern is rather sporadic with limited regularity.

6.1.2 Energy Plans. We consider the energy plans available in New York State [21]. Due to a large number of suppliers in this region, we select four representative energy retailers: Agera Energy, LLC, BlueRock Energy, InC, East Coast Power & Gas, LLC, and Public Power, LLC. These four retailers have constant cancellation fee $100 for a 12-month fixed-rate plan. We believe the results can hold similarly in other regions.

6.1.3 Electricity Prices. A seasonal data in 2017 is collected from New York state (Zip code: 10001). We use interpolation to obtain the exact price (p_t^0 and p_t^1) for each month. For the fixed-rate plan, we set H be $0.1p_t^0$, which is about the difference between rates of a variable-rate plan and a fixed-rate plan.

6.1.4 Contract Period and Cancellation Fee. We set the length of a fixed-rate plan to be 12 months, with $100 constant cancellation fee when quitting before the term ends. As for the linearly decreasing cancellation fee plan, we charge $10 to every rest month remaining in the contract, which is consistent with many settings for energy retailers, for example Kiwi Energy NY, LLC and North America Power & Gas, LLC.

6.1.5 Cost Benchmark. As discussed in Section 1, we view current consumers as stationary plan users, who do not change their original plan throughout an entire year. Since the fixed-rate plan from East Coast Power & Gas is the lowest in 2017, we assume customers sticking to the variable-rate plan offered by this retailer. We evaluate their cost reduction by applying different algorithms.

6.1.6 Comparisons of Algorithms. We compare algorithm OFA_s, gCHASE_s and gCHASE_s^r for the same application scenario. The difference is that OFA_s gets all input information before the beginning of time, but gCHASE_s and gCHASE_s^r are not feed with the current input until time arrives. For each particular setting, gCHASE_s^r runs 100 times and is judged by its average outcome.

6.2 Constant Cancellation Fee

6.2.1 Purpose. In this setting, we aim to answer two questions. First, what is the maximum saving a consumer can benefit from by applying the optimal offline algorithm? From Figure 2(c), staying in a variable-rate plan of the same retailer for long is not economic. It is then interesting to evaluate the exact gap between switching and not. Second, how well can our proposed online algorithms behave comparing to the optimal one? Offline algorithm needs full time input before time starts, which is not practical for consumers to use. On the other hand, online algorithms are more practical in the sense that they do not require any predictions or stochastic models of future inputs, but their performances need to be validated.

6.2.2 Observations. From Figure 3, we observe that OFA_s can save 16% - 18% for 50% of the families. We regard it as a large benefit, since on average it indicates that a family saves 2 months' bill in a year. The results justify importance on switching energy plans properly. Further, by comparing it with our proposed deterministic and randomized online algorithms, we verify that OFA_s is always the best. We claim that both gCHASE_s and gCHASE_s^r are competitive as it is clearly shown in the figure that 14% - 16% saving can be guaranteed for 50% of the families. We find out that our two online algorithms have roughly the same behavior. Due to the inherited randomness in gCHASE_s^r, a particular family may not be guaranteed to reap more savings by introducing the randomized online algorithm to this setting.

6.3 Linearly Decreasing Cancellation Fee

6.3.1 Purpose. For the case when cancellation fee is linearly decreasing with the time of enrollment in a fixed-rate plan, we implement the same online algorithms to cost data as in Section 6.2. However, the optimal offline solution is derived by applying dynamic programming technique, which is denoted by OFA in the legend.

In this experiment, we would like to identify the influence of changing settings, and find out whether the high percentage of savings can still be achieved or performances of algorithms will change greatly. Moreover, our conjecture in Section 5 is evaluated by checking behaviours of online algorithms.

6.3.2 Observations. We notice that the behaviour of OFA in Figure 4 is exactly the same as that of OFA_s in Figure 3. Since the total decision space is only 12 months, not introducing cancellation fee should be an optimal scheduling for the consumer. As a result, in both scenarios, the optimal offline algorithm for every family is staying in a variable-rate plan for a period, then switching to the fixed-rate plan if needed. For the deterministic online algorithm gCHASE_s, in equation (13) α is relatively small comparing to $\delta(t)$ in reality. Hence, for most families, we observe minor distinctions in their cost savings between Figure 3 and Figure 4, which means their decisions are almost unchanged. For the randomized online algorithm gCHASE_s^r, an obvious shift can be detected. The cost saving percentage by applying two online algorithms vary largely for most of the families. Additionally, there are more than 60% of the families having less cost by utilizing gCHASE_s^r rather than gCHASE_s, whereas the rest see the converse pattern. Overall, we

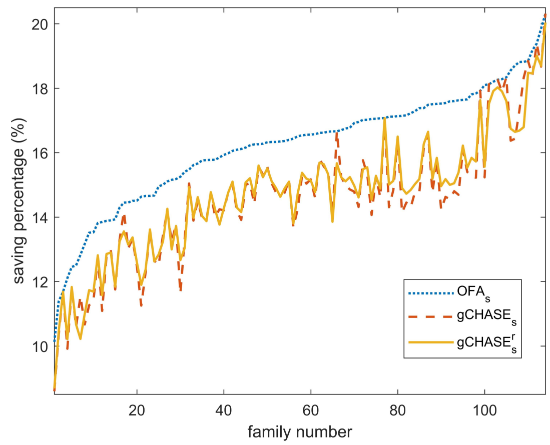

Figure 3: Constant cancellation fee – percentage of savings of three algorithms comparing to a variable-rate plan of East Coast Power & Gas.

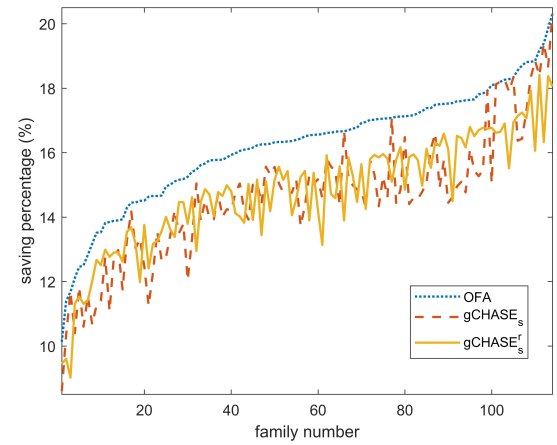

Figure 4: Linearly decreasing cancellation fee – percentage of savings of three algorithms comparing to a variable-rate plan of East Coast Power & Gas.

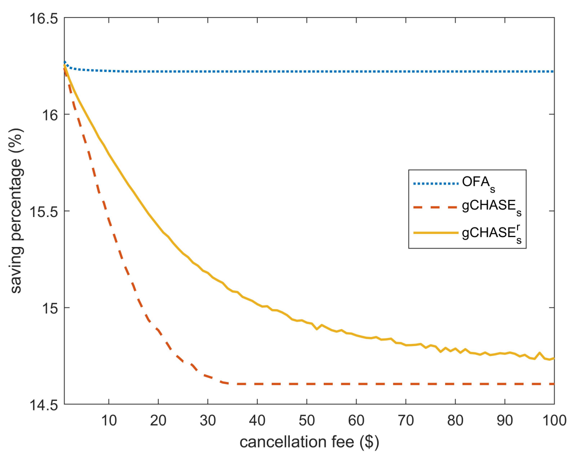

Figure 5: Cost saving percentage as a function of cancellation fee (constant over time) – comparing to a variable-rate plan of East Coast Power & Gas.

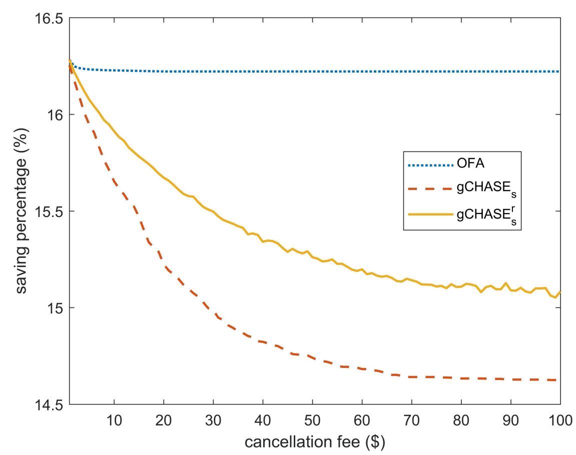

Figure 6: Cost saving percentage as a function of cancellation fee (linearly decreasing over time) – comparing to a variable-rate plan of East Coast Power & Gas.

conclude that randomization makes behaviour of the online algorithm in a larger range, thus its performance is less stable. However, it also has benefits of surpassing the deterministic one in some cases.

6.4 Effect of Cancellation Fee

6.4.1 Purpose. Previous experiments show that our proposed online algorithms are indeed competitive with large saving. This section is designed to investigate the impact of changing cancellation fee while keeping other parameters unchanged. Whether competitiveness of online algorithms can be preserved in various scenarios remains a question to be answered. Based on intuition, when cancellation fee is relatively high, consumers tend to be inactive without taking extra thoughts for choosing to enroll or quit a fixed-rate plan as they do not tend to suffer huge punishment. Our goal is to find out if consumers still benefit from switching, and the gap between online algorithms and the offline optimal one. Ultimately, we long for guiding consumers on properly choosing their energy plans.

To increase chances of switching between plans, we let cancellation fee β vary from \$1 to \$100, with \$1 increment between each two trials. To understand its different influences under both constant and linearly decreasing cancellation fee settings, we conduct two experiments for them separately. Figure 5 plots the average cost saving percentage of all families under 100 distinct constant cancellation fee settings. As for the linearly decreasing cancellation fee case, we divide β by length L to get α in each trial, and results are shown in Figure 6.

6.4.2 Observations. First of all, two figures generally reveal same trends for these algorithms. With the growth of cancellation fee, saving percentage decreases. The offline algorithm is relatively unchanged, due to the reason that for most of the times, it simply chooses to change from the variable-rate plan to the fixed-rate plan without incurring any additional cost. However, for gCHASE$_s$ and gCHASE$_s^r$, when the cancellation fee takes large value, the impact of making a wrong decision incurs higher penalty, thus its competitiveness decreases.

Second, the plot indicates that by decreasing cancellation fee, online algorithms turn to be more competitive. Overall, the randomized online algorithm has better behavior than the deterministic one, where the advantage of making decision randomly is revealed. It seems that all algorithms reach 'saturated' saving percentage with the growth of cancellation fee. Moreover, by viewing from a larger angle, we notice that although the cancellation fee has high variation, the saving percentage only differs by around 1.5%. These results indicates that the most saving comes from choosing the cheapest plan for all the time, and online algorithms will not get too bad even when cancellation fee is high.

Third, by comparing Figure 5 and Figure 6, we observe that online algorithms achieve better behaviour in the linearly decreasing cancellation fee setting. It can be interpreted by considering that even if a wrong decision is made, for most circumstances, only partial of the maximum cancellation fee β is needed to be charged. Further, randomization has more advantage in the linearly decreasing cancellation fee setting, although it is an average result for all households instead of each individual case.

6.5 To Stay or to Switch

Thus far, we have examined the performance of our algorithms in both settings of constant and linearly decreasing cancellation fee, and under various scenarios of cancellation fee. In practice, how and when should a household choose to stay in or switch to an energy plan depends on several critical factors as follows:

- Among a large number of variable-rate plans provided by retailers, consumers should always find the one who provides with the lowest rate for the current month.
- For all available fixed-rate plans, they should first be ranked from the lowest to the highest by average monthly rate throughout a year. Four criteria are to facilitate consumers' decisions: (1) low rate rank, (2) low cancellation fee (average $ / month), and (3) linearly decreasing setting.
- If possible, try to implement a randomized online algorithm, especially for the linear decreasing cancellation fee scenario.

Following the above tips, a household can save up to 15% energy cost on average in our evaluation.

7 RELATED WORK

The problem formulation studied in this paper is closely related to the subjects of Metrical Task System problem [8] and online convex optimization problem with switching cost [6].

When the state space belongs to a discrete set, such an online decision problem is known as Metrical Task System (MTS) problem, whose competitive ratio is known to be $2n - 1$ [9] in a general setting with n states. Recently, [17] studied the energy generation scheduling in microgrids, which belongs to a subclass of MTS problems with convex objective function and linear switching cost. An online algorithm is developed called CHASE and can achieve a competitive ratio of 3, which is the lower bound for any deterministic online algorithms for this problem. Remarkably, restricting to convex operational cost functions and linear switching cost can reduce the optimal competitive ratio (termed as price of uncertainty) for MTS problems from $2n - 1$ to a constant 3.

On the other hand, an online convex optimization problem with switching cost is similar to an MTS problem, except that the state space is a continuous set. A 3-competitive algorithm called LCP is developed in [15] for it. Recently, [3] shows that the same competitive ratio can be maintained in the discrete setting under proper rounding. Furthermore, it is proved that the competitive ratio of 3 is the lower bound of all deterministic online algorithms in the discrete setting.

Moreover, [3] and [4] show that by applying proper randomization, a 2-competitive online algorithm can be constructed based on [5]. In this paper, we combine these two papers and construct an uniform version of algorithms, which is more practical and easier to be implemented.

Finally, a key difference from the previous studies that makes our problem harder is the presence of temporally dependent switching cost. To the best of our knowledge, this work appears to be the first study considering this class of problems.

8 CONCLUSION AND FUTURE WORK

This paper presents effective online decision algorithms to assist the energy plan selection in a competitive energy retail market to save energy cost. We devised offline optimal and competitive online algorithms. For the case of constant switching cost, we characterized the competitive ratios of our deterministic and randomized online algorithms and proved that they are the best possible in their classes. For a more general case with linearly decreasing switching cost, we developed a heuristic online algorithm and conjectured its performance. Empirical evaluations based on real-world data traces corroborated the effectiveness of our algorithms and demonstrated that opportunistic switching among energy plans can indeed bring considerable savings in energy cost to average customers.

As for the future work, we plan to evaluate the effectiveness of our algorithms with more diverse real-world data traces. Also, it is theoretically important and practically useful to develop more general online algorithms with improved competitive ratios for metrical task system problems with arbitrarily temporally dependent switching cost. Such an extension will be integral in solving a more generic class of energy plan selection problems, as well as online decision problems in broad applications.

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