Competitive Online Age-of-Information Optimization for Energy Harvesting Systems

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***Abstract*—We consider the scenario where an energy harvesting source sends its updates to a receiver. The source optimizes its energy allocation over a decision period to maximize a sum of time-varying functions of the age of information (AoI), representing the value of providing timely information. In a practical online setting, we need to make irrevocable energy allocation decisions at each time while the time-varying functions and the energy arrivals are only revealed sequentially. The problem is then challenging as 1) we are facing uncertain energy harvesting arrivals and time-varying functions, and 2) the energy allocation decisions and the energy harvesting process are coupled due to the capacity-limited battery. In this paper, we develop an optimal online algorithm CR-Reserve and show it achieves $(\ln \theta + 1)$-competitive, where θ is a parameter representing the level of uncertainty of the time-varying functions. It is the optimal competitive ratio among all deterministic and randomized online algorithms. We conduct simulations based on real-world traces and compare our algorithms with conceivable alternatives. The results show that our algorithms achieve 12% performance improvement as compared to the state-of-the-art baseline.**

I. Introduction

In many real-time systems, e.g., sensing, monitoring, mobility tracking, and networked control, obtaining timely updates is crucial for ensuring the performance of the systems. Age of Information (AoI) is a widely adopted measure of the timeliness of the information on the receiver side. It measures the elapsed time since the last received sensing update was generated. Typically, maintaining a low AoI ensures that the receiver has more accurate information about the sensing objectives and leads to better system performance.

When the source of the updates is powered by an energy-harvesting battery, it can not provide updates at all times. To optimize AoI, we need to carefully schedule the update with respect to the available energy. However, the energy harvesting arrival is dynamic in nature and hard to predict [1]. If we are too optimistic about the uncertain energy arrival, we may run out of energy too early and experience overly outdated information on the receiver. In contrast, being too conservative, we may miss some energy harvesting arrivals due to the capacity limit of the battery. Thus, it calls for a robust solution to optimize energy utilization under arbitrary energy harvesting arrivals.

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Moreover, in practice, the relationship between the system performance and the age of information is usually non-linear [2] and could even be time-varying [3], [4]. Simply optimizing the linear AoI may not be efficient enough to guarantee the overall system performance. For example, in the mobility tracking problem [4], the real-time tracking error is determined by the age of information and the highly non-stationary motion of the moving object. It is crucial to consider optimizing the update scheduling under uncertain and time-varying functions of AoI that capture the real-time relationships between the system performance and AoI.

While there have been existing studies on AoI optimization for energy harvesting systems, they consider either long-term average AoI or known fixed function of AoI, e.g. [5], [6]. Moreover, they assume an independent and identically distributed (i.i.d.) energy harvesting process. Such an assumption can hardly be true in practice as we also observe in the real-world traces (see Fig. 4 in Sec. V). Also, the empirical performance could degrade significantly in real-world traces compared with that under the assumption (see Fig. 5 in Sec. V). It remains open to optimizing the time-varying functions of AoI for an energy harvesting system without assuming i.i.d. energy arrivals. We defer a detailed discussion on related work to Sec. II.

In this work, we propose a competitive online optimization approach for the problem. In an online manner, the time-varying functions of AoI and energy harvesting arrivals are only revealed sequentially. One needs to make irrevocable decisions at each time without the information of future input. We consider arbitrary energy harvesting arrivals without any assumption. We aim to design competitive online algorithms that, regardless of the input, achieve a close performance to the offline optimal that knows the whole input in advance.

In this paper, we study the online AoI optimization problem for an energy harvesting system. We consider maximizing aggregated time-varying values of maintaining a low AoI by optimizing the energy allocation over a decision period. The online problem is challenging as we face uncertain energy harvesting arrivals and time-varying value functions that are only revealed sequentially. Moreover, there are strong couplings between the energy allocation and harvesting decisions. The energy allocation at the current moment would affect the future energy allocation due to limited available energy and the future energy harvesting due to the limited battery capacity. Despite these challenges, we carry out a comprehensive study of the problem and make the following contributions.

▷ We propose the problem of maximizing a sum of time-

TABLE I: Summary of existing studies and this work.

	Objective	Capacity Limit	Energy Harvesting	Time-Varying Objective	Uncertainty (Objective)	Uncertainty (Harvesting)	Performance Metric	Optimality
[7]	AoI	✗	✓	✗	N.A.	Stochastic	Expected Value	✓
[5], [6]	AoI	✓	✓	✗	N.A.	Stochastic	Expected Value	✓
[4]	AoI	✗	✗	✓	Arbitrary	N.A.	Regret	Order Opt.
[8]	General	✓	✓	✓	Arbitrary	Stochastic	Regret	Order Opt.
[1]	Throughput	✓	✓	✓	Arbitrary	Arbitrary	Competitive Ratio	Order Opt.
[9], [10]	General	✓	✗	✓	Arbitrary	N.A.	Competitive Ratio	✓
This work	General	✓	✓	✓	Arbitrary	Arbitrary	Competitive Ratio	✓

varying values of maintaining a low AoI for energy harvesting systems and formulate the problem under an epoch-based setting in Sec. III. We are the first to consider competitive online AoI optimization for energy harvesting systems with time-varying objectives and arbitrary energy harvesting arrivals.

▷ In Sec. IV, we propose an online algorithm, named CR-Reserve, and show that it achieves the optimal competitive ratio of $\ln\theta + 1$, where θ is the parameter capturing the uncertainty level of the value of timely information. Our design introduces a novel idea of reserving enough energy to maintain the worst-case performance guarantee under arbitrary future input while greedily allocating the remaining energy to exploit the non-worst-case input. In addition, we generalize an existing algorithm design framework CR-Pursuit [9], [10] to involve energy harvesting with a capacity-limited battery. With the generalization, we can then determine the worst-case performance guarantee and the amount of energy to reserve for the worst case. Further, compared with the framework, our algorithm further exploits the non-worst-case input and achieves a preferable empirical performance while maintaining the worst-case performance guarantee. Our CR-Reserve providing a novel design idea and optimal solutions for a general class of online optimization problems with inventory capacity constraints and inventory replenishment could be of independent interest.

▷ In Sec. V, we conduct simulations to evaluate the empirical performance of our proposed algorithm. We show that our algorithms outperform the state-of-the-art baseline under various settings in real-world traces and achieve an 12% performance improvement under typical settings.

II. Related work

Optimizing information timeliness (measured by the age of information, AoI) for energy harvesting systems has been widely studied in the literature, e.g., [7], [11], [12], [13], [14], [15], [16], [6], [17], [5], [18]. Different models of the energy harvesting systems and the energy harvesting process are considered. In [7], the authors consider that the average power consumption is limited by the energy harvesting rate without involving the battery capacity limit. The authors in [19] show the optimal policy under the case of a capacity-one battery and an infinite-capacity battery. In [6], [5], the authors further show the optimal policy under a finite capacity battery. In these solutions, the optimal solutions follow a threshold-base structure. That is to update when the current AoI exceeds a threshold (which may depend on the state of the battery). In [11], the authors propose the offline optimal solution with full information on the energy harvesting input. The other studies are based on the assumption of independent and identically distributed energy harvesting processes and aim to minimize the expected value, including average AoI or functions of AoI. As a comparison, we consider a capacity-limited battery and do not assume any information about the energy harvesting process. Further, we consider arbitrary time-varying objectives in this work. Instead of focusing on the expected performance, we aim to design optimal online algorithms without relying on any stochastic information or knowledge of the future objective function and provide worst-case performance guarantees against the offline optimal with full knowledge of the input.

There has been substantial research on optimizing energy harvesting systems for various objectives, e.g., throughput maximization [1], sensing performance [20], utility maximization [21], [22], federate learning [23] and etc. These studies mostly consider i.i.d energy harvesting process and known time-invariant objectives. In this work, we provide an online competitive analysis on the value of timely information maximization problem under adversary value functions and energy harvesting processes. Among them, one particularly related study is [1] where the authors propose an order-wise optimal online algorithm for the throughput maximization problem under time-varying channel gain and arbitrary energy harvesting processes. However, this order-wise optimal online algorithm exploits the particular form of the objective function. We note that our model, formulation, and solutions are much more general, and we propose a simple online algorithm that achieves the exact optimal competitive ratio.

In [4], the authors propose an epoch-based formulation for minimizing the time-varying cost of AoI, which also inspires our epoch-based settings. However, they do not involve energy harvesting systems in the model, and their results can not be directly applied due to the hard energy constraints. In our work, we consider a source device with a capacity-limited battery under which the information update process is subject to hard energy constraints. We note that the battery model is not only practically relevant but also theoretically interesting, as we discussed in Sec. I.

In online optimization, our problem belongs to the class of competitive online optimization under inventory constraints, e.g., [24], [9], [10], [25], [26], [27]. Our work generalizes [9], [10], [25] by involving inventory replenishment (cf. energy harvesting) and storage capacity (cf. battery capacity). We consider general concave objective functions whereas only linear functions are involved in [27], [26]. In the literature,

TABLE II: Key notations.

Notation	Definition
T	Number of time epochs
$g_t(\cdot)$	Value function at epoch t
L, U	Lower bound, upper bound on the gradient of $g_t(\cdot)$
θ	U/L, uncertainty level of the value functions
C	Battery capacity and initial stored energy
$\hat{h}_t$	The energy arrival at epoch t
h_t	The energy harvesting at epoch t, $h_t \leq \hat{h}_t$
β	The energy allocation limit at epoch t
v_t	The energy allocation decision at epoch t, $v_t \leq \beta$.

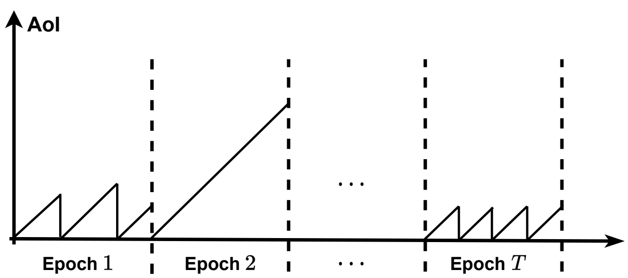


Fig. 1: An illustration of the epoch-based setting.

online learning is another widely applied approach for handling uncertain and possibly adversarial input, e.g. AoI minimization [4] and optimizing energy harvesting systems [8], [28]. In online learning, the online performance is measured by regret, which is the performance difference compared with the offline optimal restricted to fixed policies [8], [4] or bounded dynamic polices [4]. Also, online learning focuses on asymptotic performance guarantees as the decision period approaches infinity. In competitive online optimization and this work, we compare the relative performance of an online algorithm with the exact offline optimal without those restrictions and provide performance guarantees that hold at any time. We further note that for [8], [28] working on energy harvesting systems, it is crucial for them to assume i.i.d energy harvesting arrivals, without which there are no available results.

We summarize the most related studies and our work in Table I. Overall, we are the first to consider the general online AoI optimization problem for energy harvesting systems with arbitrary energy harvesting arrivals and provide the exact optimal competitive ratio.

III. System Model and Problem Formulation

We consider that an energy harvesting source sends updates to a receiver in a decision period. Our goal is to maximize the value of maintaining a low AoI by optimizing the energy allocation among the epochs. At each epoch, given the allocated energy, we optimize the update schedules to provide timely information to the receiver, while the incurred value (e.g., improved system performance with timely information) is time-varying across different epochs.

A. System Model

We consider an epoch-based scenario [4] that the entire decision period is divided evenly into multiple epochs. In each epoch, we determine the update of the source to optimize the value of maintaining a low AoI, which can vary among different epochs as appearing in many practical applications [3], [4], [29]. For example, in the vehicular network, the timely information of the monitored vehicle becomes more critical when it is passing an intersection or overtaking [29]. We will formally introduce the time-varying value of maintaining a low AoI in an epoch later. We consider zero service times, i.e., the update arrives at the receiver instantly [5], [6]. We consider that the AoI is reset to zero at the end of each epoch. This allows us to consider the AoI process in each epoch separately and define the value of maintaining a low AoI in an epoch without considering the interference from its previous epoch. However, we need to allocate the energy to support the updates in each epoch, and the energy allocation decisions are subject to the battery capacity limit and the energy harvesting process. In practice, we can require an update at the end of each epoch to reset the AoI. We provide an illustration of the epoch-based setting in Fig. 1.

Energy Harvesting Systems. We denote the capacity of the battery in the source as C. Without loss of generality, we consider that the battery is fully charged at the beginning. We denote by v_t the energy allocation for sending updates at epoch t, which is subject to a rate limit denoted as β. Suppose the amount of energy arrival at epoch t is $\hat{h}_t$. Due to the capacity limit, there may be a battery overflow preventing from harvesting the entire energy arrival. We denote the amount of harvested energy (energy charged to the battery) at epoch t as h_t, which is limited by $\hat{h}_t$. We assume that the energy harvested in an epoch can be directly used in the current epoch. Then we can compute the stored energy at the end of epoch t as $C - \sum_{\tau=1}^{t} v_\tau + \sum_{\tau=1}^{t} h_\tau$, which should be within $[0, C]$.

AoI Optimization. Suppose there are T epochs, each with length $\hat{\tau}$. Without any update, the peak AoI at each epoch equals $\hat{\tau}$ (the maximum AoI in the epoch). With the allocated energy v_t in epoch t, we can optimize the update schedule in the epoch, and we denote the optimized peak AoI at epoch t as $a_t(v_t)$. We model the value of maintaining the AoI $a_t(v_t)$ at epoch t as a concave and increasing function of reduction of peak AoI, which we denote as $f_t(\hat{\tau} - a_t(v_t))$. We denote such value with regard to the energy allocation decision as $g_t(v_t) = f_t(\hat{\tau} - a_t(v_t))$.

We consider that $a_t(v_t)$ is a decreasing and convex function of v_t. For example, if we consider that each update consumes η unit of energy [5], [6], then given the allocated energy v_t, we could show that the optimal solution for maximizing the value of timely information is to send the update evenly and the resulted peak AoI a_t is approximately $\hat{\tau}/(v_t/\eta + 1)$.

Overall, we have that $g_t(v_t)$ is an increasing and concave function regarding the energy allocation v_t at epoch t. We assume that $L \leq g_t'(v_t) \leq U, \forall v_t, t$, i.e., the marginal value of the energy allocation is always within $[L, U]$. We define $\theta = U/L$, which represents the level of uncertainty of the value functions. And $g_t(0) = 0$, i.e., there is no benefit in not allocating energy. We provide a concrete example of the value functions in a mobility tracking application in the simulation.

We note that while we only discuss peak AoI and assume zero service time in our illustration of AoI optimization, our approach can be applied to consider average AoI or other more general metrics related to AoI with different service time models. Our approach only requires the induced value function $g_t(v_t)$ regarding the energy allocation decision to satisfy the conditions discussed above.

B. Problem Formulation

To this end, we formulate the general AoI optimization (GAoI) problem as follows,

$$\textsf{GAoI} \quad \max \quad \sum_{t=1}^{T} g_t(v_t) \tag{1}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{t} v_\tau \leq C + \sum_{\tau=1}^{t} h_\tau, \forall t \in [T] \tag{2}$$

$$\sum_{\tau=1}^{t} v_\tau - \sum_{\tau=1}^{t} h_\tau \geq 0, \forall t \in [T] \tag{3}$$

$$\text{var.} \quad 0 \leq v_t \leq \beta, 0 \leq h_t \leq \hat{h}_t, \forall t \in [T]. \tag{4}$$

In GAoI, our goal is to maximize the value of maintaining a low AoI by optimizing the energy allocation and energy harvesting decisions. Our decisions are subject to the following constraints. The energy allocation constraints (2) require that the total energy allocation is bounded by the sum of the initial SoC and the total harvested energy. The energy harvesting constraints (3) mean that harvested energy should not exceed the allocated energy. It is derived from the battery capacity constraint that the stored energy should not exceed the battery capacity at all slots, i.e., $C + \sum_{\tau=1}^{t} v_\tau - \sum_{\tau=1}^{t} h_\tau \leq C$. Our decisions at each epoch are also subject to the energy allocation rate limit and energy harvesting limit.

In practice, the source harvests energy as much as possible until the battery is fully charged. There are no other energy harvesting decisions. We note that our formulation also matches such a practical scenario under the optimal solutions. By introducing energy harvesting decisions, our modeling only involves linear constraints. It also facilitates discussions on our algorithm design and performance analysis.

In addition, depending on the application, there could be specific value functions of maintaining AoI, i.e., $f_t(\cdot)$ and the resulting $g_t(\cdot)$. It is possible to adapt our approach and further explore application-specific structures. However, as the first study to consider competitive online AoI optimization, we focus on a general set of functions and would like to leave the specific applications for future study.

C. Online Scenario and Challenges

We note that in the offline setting where all the input are known in advance, GAoI is a simple convex optimization problem with linear constraints where efficient solutions exist [30]. However, in practice, we are facing an online scenario where the input are revealed sequentially, and we need to make irrevocable decisions after each revelation. In this paper, we focus on the online setting and apply the *competitive ratio* (CR) as the performance metric of an online algorithm. The CR of an algorithm $\mathcal{A}$ is defined as,

$$\mathcal{CR}(\mathcal{A}) = \sup_{\sigma \in \Sigma} \frac{OPT(\sigma)}{ALG(\sigma)}, \tag{5}$$

where σ denotes an possible input sequence and Σ represents all possible input. We use $OPT(\sigma)$ and $ALG(\sigma)$ to denote the offline optimal objective and the online objective of algorithm $\mathcal{A}$ under the input σ, respectively. We do not assume a given decision period T, i.e., σ could be input of arbitrary lengths. An online algorithm is called α-competitive if $\mathcal{CR}(\mathcal{A}) \leq \alpha$.

In the competitive analysis, we focus on the worst-case guarantee of an online algorithm, which is defined by the maximum performance ratio between the offline optimal and the online objective of the algorithm. In the online setting, the problem is challenging due to 1) allocating limited energy over a period with time-varying and partially revealed value functions [9], [24] and 2) the limited amount of energy, which implies that allocating more energy may lead to an energy shortage in the future epochs [9], and 3) the battery capacity limit, which means that a conservative energy allocation may miss future energy arrivals [1]. Our problem can be viewed as a general kind of online optimization problem under inventory constraints with inventory replenishment (cf. energy harvesting) and storage capacity limit (cf. battery capacity). It generalizes the existing studies where only a static inventory is considered [9], [10] (cf. no energy harvesting). Involving energy harvesting with battery capacity limit is non-trivial compared with [9], [10] due to the third challenge. In [1], the authors design a specific solution to handle the third challenge (they call it the battery overflow problem), which incurs a non-small constant factor on the competitive ratio. Here, by further exploring the problem structure, we propose an online algorithm CR-Reserve that achieves the same and optimal competitive ratio as the case without energy harvesting or capacity limit, which we will discuss next.

IV. Online Algorithm and Performance Analysis

In the section, we propose an optimal online algorithm for the online problem GAoI, named CR-Reserve. We first give an overview of the algorithm. We then describe the algorithm in detail and show it achieves the optimal competitive ratio.

A. Overview of the Algorithm Design

The idea of CR-Reserve is to reserve enough energy for future input to maintain the worst-case performance guarantee while greedily allocating the remaining energy to exploit the non-worst-case input for preferable empirical performance. To determine the achievable worst-case performance guarantee (or competitive ratio) and amount of energy to reserve at each epoch, we generalize an existing algorithm design framework CR-Pursuit [9], [10] dedicated to the worst-case analysis and restricted to the non-energy-harvesting scenario.

The CR-Pursuit framework follows a neat idea that at each time, it determines the online decision to maintain a constant performance ratio (say π) between the offline optimal given the input up to the current time and the current online performance. Naturally, if the resulting online decisions always exist and are feasible, we achieve a competitive ratio of π. While the idea of CR-Pursuit is simple, it is usually non-trivial to determine the optimal parameter π to pursue and provide the optimality analysis of the derived algorithm. It requires one to exploit the problem structure, especially in identifying the worst-case input, which is mostly specific to each particular online optimization problem, e.g., [9], [24], [31], [10], [32], [33].

In terms of our problem GAoI, we note that existing studies only cover the case without energy harvesting [9], [10], i.e., when $h_t = \hat{h}_t = 0, \forall t$. Meanwhile, the consideration of energy harvesting with limited battery capacity introduces new and fundamental challenges in determining the optimal π. First, we face a dynamic amount of total energy to allocate due to energy harvesting. As a comparison, we only need to guarantee the choice of π satisfying a single fixed total energy allocation constraint when without energy harvesting. Second, with the battery capacity limit, the energy allocation now would affect energy harvesting in future epochs. Moreover, trying to save energy for potentially better value functions may, in turn, lead to a loss of energy harvesting. We resolve these new challenges and determine the optimal competitive ratio for GAoI.

CR-Pursuit focuses on the worst-case analysis and maintains the worst-case ratio regardless of the input. Such conservative behavior may not favor empirical performance where the worst case seldom happens. However, it provides an upper bound on the energy needed to reserve for maintaining the worst-case ratio under arbitrary future input. Our online algorithm CR-Reserve then greedily allocates the remaining energy to exploit the non-worst-case input, especially when the revealed values are high, leaving less uncertainty in the future. We show that CR-Reserve could preserve the optimal worst-case performance guarantee while achieving preferable empirical performance.

Next, we will first follow the CR-pursuit framework and determine the optimal competitive ratio in Sec. IV-B. We will present CR-Reserve in Sec. IV-C. We provide an illustration of our algorithm in Fig. 2.

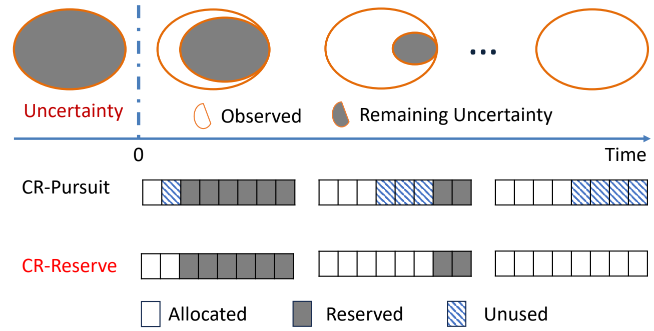


Fig. 2: An illustration of CR-Reserve. For ease of representation, we assume no energy arrivals and that the energy allocations happen to be in units. With the input being revealed, the remaining uncertainty set shrinks. CR-Reserve computes the energy needed to reserve and greedily allocate the remaining energy. In contrast, CR-Pursuit only looks at the revealed input and allocates the needed energy to maintain the worst-case ratio (note that it does not compute the reserved energy and unused one, which are marked for illustration only). It results in unused energy and restricted performance when the uncertainty set shrinks fast under a non-worst case.

B. Determine the Optimal Competitive Ratio

We now describe the CR-Pursuit(π) algorithm to online GAoI, where π is a parameter to be specified. We denote by OPT_t the optimal solution to GAoI given the input up to epoch t. We set $OPT_0 = 0$. At each epoch t, we determine the energy allocation $\tilde{v}_t$ such that

$$g_t(\tilde{v}_t) = \frac{1}{\pi}\left(OPT_t - OPT_{t-1}\right). \tag{6}$$

We harvest all the energy arrival subject to the battery capacity, i.e., we determine the energy harvesting $\tilde{h}_t$ such that

$$\tilde{h}_t = \max\left\{\hat{h}_t, \sum_{\tau=1}^{t}\tilde{v}_\tau - \sum_{\tau=1}^{t-1}\tilde{h}_\tau\right\}. \tag{7}$$

Following (6), we could see that if its output is feasible under any possible input, we naturally attain a π-competitive solution. Also, we could see that by choosing the energy as (7), it always satisfies the energy harvesting constraints (3) and the harvesting limits $\hat{h}_t, \forall t$. It leads to the critical design of CR-Pursuit to GAoI - how to determine the minimum π such that CR-Pursuit(π) always satisfies the energy constraints (2).

We first summarize our results in the following theorem that by choosing $\pi = \ln\theta + 1$, CR-Pursuit achieves the competitive ratio $\ln\theta + 1$, and it is the optimal one for GAoI.

Theorem 1. *CR-Pursuit($\ln\theta + 1$) is ($\ln\theta + 1$)-competitive. And it is optimal among all online algorithms for GAoI.*

We discuss our idea here and leave the proof in Appendix A. We first show in Lemma 3 (Appendix A) through divide-and-conquer that when the battery has an unlimited battery capacity, i.e., both the online algorithm and the offline algorithm can harvest all the energy arrival, CR-Pursuit can achieve the competitive ratio $\ln\theta + 1$. This resolves the first challenge discussed in Sec. IV-A, i.e., dynamic total available energy.

Then, when considering a capacity-limited battery, it is more challenging for the online algorithm that the conservative energy allocation of CR-Pursuit, i.e., following (6), may lead to less empty capacity for future energy harvesting. Consequently, CR-Pursuit may harvest less energy compared with the offline optimal and could fail to maintain the same competitive ratio $\ln\theta + 1$ with a less amount of energy. We show that CR-Pursuit can handle such a challenge by critical observations as follows. The offline optimal is also subject to the battery capacity limit, i.e., the optimal solution should satisfy (3). That is, it should at least allocate the same amount of energy as the harvested energy at the precedent epochs, which could not be re-optimized at the later epochs. Beginning from the next epoch, the offline optimal could only re-allocate an amount of energy of the capacity limit and the newly harvested energy. Meanwhile, when CR-Pursuit harvests less energy, the battery is fully charged (as it is initially). It is like returning to the initial state of the problem, under which we can show that the allocation of CR-Pursuit is bounded by that under the non-capacity limit case. And, it remains feasible for CR-Pursuit to pursue $\ln\theta+1$.

Finally, as GAoI covers GAoI without energy harvesting as a special case, the optimal competitive ratio for GAoI without energy harvesting, $\ln\theta+1$ (as shown in Theorem 5 of [10]), is naturally a lower bound for the optimal competitive ratio of GAoI. Thus, CR-Pursuit$(\ln\theta+1)$ achieves the optimal competitive ratio for GAoI.

C. Algorithm CR-Reserve

We now introduce our newly proposed online algorithm CR-Reserve. In CR-Reserve, at each epoch, given the observed input, we compute an upper bound on the minimum required energy for maintaining the optimal competitive ratio under arbitrary future input following the worst-case analysis and CR-Pursuit. We then either greedily allocate the energy once the remaining energy is no less than the upper bound or follow CR-Pursuit, depending on which one allocates more energy. We summarize the algorithm in Algorithm 1.

In more detail, at each epoch t, we denote the stored energy at the beginning of slot t as $b_t = C - \sum_{\tau=1}^{t-1}\left(\tilde{v}_\tau^G - \tilde{h}_\tau^G\right)$, where $\tilde{v}_\tau^G$ and $\tilde{h}_\tau^G$ are previous energy allocation and harvesting decisions of the algorithm. We derive an upper bound of minimum energy required for CR-Pursuit to maintain $\ln\theta+1$ in the future epoch,

$$\hat{\Phi}_t = \frac{\ln\left(U/\max\{\lambda_t, L\}\right)+1}{\ln\theta+1}\cdot C, \tag{8}$$

where λ_t is the optimal dual variable associate with the energy allocation constraint (2) at epoch t in OPT_t. Then, we determine the greedy energy allocation, $\hat{v}_t$, as the optimal value of the following simple linear programming

$$\max_{v_t, h_t} v_t \tag{9}$$

$$\text{s.t } \hat{\Phi}_t \le b_t - v_t + h_t \le C \tag{10}$$

$$v_t \le \beta, h_t \le \hat{h}_t. \tag{11}$$

And we determine the online allocation $\tilde{v}_t^G$ as $\max\{\hat{v}_t, \tilde{v}_t\}$. We determined the energy harvesting $\tilde{h}_t^G$ similar as (7).

We then discuss the idea of obtaining the upper bound on the minimum required energy $\hat{\Phi}_t$. To compute $\hat{\Phi}_t$, we can use the conservative algorithm CR-Pursuit$(\ln\theta+1)$ as a baseline. That is, we compute the maximum energy CR-Pursuit$(\ln\theta+1)$ requires under arbitrary future input.

We first consider the case that there is no future harvesting. At epoch t, we can compute the required remaining energy running CR-Pursuit$(\ln\theta+1)$ as

$$\Phi_t = \sup \sum_{\tau=t+1}^{T} v_\tau \tag{12}$$

$$\text{s.t. } g_t(v_\tau) = \frac{OPT_\tau - OPT_{\tau-1}}{\ln\theta+1}, \forall t+1 \le \tau \le T \tag{13}$$

$$\text{var. } T, g_t(\cdot), \forall t+1 \le \tau \le T. \tag{14}$$

Algorithm 1: CR-Reserve

1: At epoch t, compute the offline optimal of GAoI under the input up to the epoch t,
2: Obtain $\tilde{v}_t$ of CR-Pursuit$(\ln\theta+1)$ according to (6),
3: Obtain λ_t as the optimal dual variable associated with the energy allocation constraint (2) at epoch t,
4: Obtain $\hat{\Phi}_t$ according to (8),
5: Obtain $\hat{v}_t$ by solving (9).
6: Output $\tilde{v}_t^G = \max\{\hat{v}_t, \tilde{v}_t\}$.
7: Output $\tilde{h}_t^G = \max\left\{\hat{h}_t, \sum_{\tau=1}^{t}\tilde{v}_\tau^G - \sum_{\tau=1}^{t-1}\tilde{h}_\tau^G\right\}$.

Directly computing Φ_t is complicated. However, we observe that, for all the future input, only when the gradient of the value function is larger than λ_t would the offline optimal increase and CR-Pursuit need to allocate energy according to (6) or (13). That is, the uncertainty range of the value function shrinks to $[\max\{\lambda_t, L\}, U]$. In addition, as we discuss in Sec. IV-B, the past energy harvesting can not be re-optimize in the later epoch, and thus we only need to count the increment in offline optimal due to the initially stored energy C. In CR-Pursuit for the non-harvesting case [9], [10], we need to find π such that the maximum required energy is bounded by the initial C. Here, we compute the maximum required energy given $\pi = \ln\theta+1$ but with a shrink uncertainty set. We can then follow a similar analysis to conclude an upper bound as $\hat{\Phi}_t$ in (8).

As for the case that there are future energy arrivals for harvesting, we note that CR-Pursuit either harvests all the energy arrival (with enough capacity to store) or becomes fully charged. In the first case, CR-Pursuit harvests no smaller than the offline optimal and thus could maintain the ratio $\ln\theta+1$ as we show in Lemma 3 (Appendix A). In the second case, the CR-Pursuit is with a fully charged battery again and following the similar idea as we showed Theorem 1, CR-Pursuit can maintain $\ln\theta+1$ for all future input at such a case.

Further, as we apply the upper bound on the minimum required energy, it may lead to a limited allocation at the current epoch. To maintain the ratio $\ln\theta+1$, we allocate no smaller the allocation CR-Pursuit$(\ln\theta+1)$ at the current epoch. Overall, we determine the energy allocation of CR-Reserve as $\tilde{v}_t^G$ in Line 6 in Algorithm 1.

With the above discussions, we can show that while the CR-Reserve tends to allocate energy more aggressively than CR-Pursuit does, it either reserves enough energy to maintain the worst-cast performance guarantee or follows CR-Pursuit$(\ln\theta+1)$. Consequently, CR-Reserve can achieve the same optimal competitive ratio as CR-Pursuit$(\ln\theta+1)$, which we summarize in the following theorem.

Theorem 2. *CR-Reserve is $(\ln\theta+1)$-competitive. And it is optimal among all online algorithms for GAoI.*

CR-Reserve achieves the optimal competitive ratio. Further, compared with CR-Pursuit, CR-Reserve only reserves enough energy for the worst case and greedily allocates the remaining energy, leading to more active utilization of the

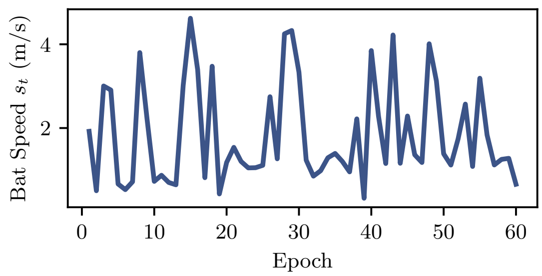


Fig. 3: Bat speed with respect to epoch. It demonstrates the fluctuating nature of the system, which leads to uncertain and time-varying value functions.

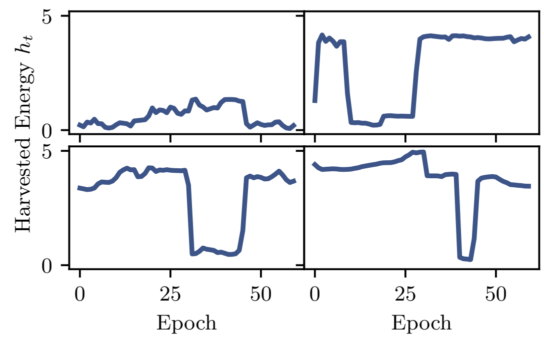


Fig. 4: Harvested energy with respect to epoch under four different instances. They follow different patterns and deviate from the i.i.d. assumption.

energy and preferable empirical performance, as illustrated in Fig. 2 and shown in the simulation.

Remark. We note that our result depends on θ, which characterizes the level of uncertainty of the input and is determined by the specific application scenarios interested. In practice, we can estimate the value of θ using domain knowledge or historical data (see an example in our simulation). We leave it as future work to analyze the impact of the estimation error of θ on the performance of online algorithms.

V. Performance Evaluation

In this section, we conduct simulations on a mobility tracking problem with real-world traces. We intend to study the effectiveness of our algorithm as compared to the conceivable alternatives under diverse settings. Also, we are interested in how our algorithm compares with the offline optimal in terms of empirical performance ratio.

A. Experimental Setup

Mobility Tracking Problem. We consider a scenario where we want to track the location of a moving object with a GPS sensor that can harvest solar energy. At any epoch t, the sensor sends updates of its real-time location to a central base station (BS). Suppose the duration of each epoch is $\hat{\tau}$ and the energy consumption per update is η, then with the amount of allocated energy v_t, we can roughly transmit $r_t = v_t/\eta$ times in epoch t. We have that $\hat{\tau}$ is the peak AoI of an epoch without any intermediate update and $\frac{\hat{\tau}}{r_t+1}$ is the peak AoI with r_t intermediate transmissions. The difference between those two terms can be treated as the reduction of the peak AoI.

In this scenario, we define the value of maintaining a low AoI $g_t(v_t)$ as the reduced maximum discrepancy between the received location update and the actual location of the object [4]. That is, we have

$$g_t(v_t) = \left(\hat{\tau} - \frac{\hat{\tau}}{r_t+1}\right) \cdot s_t = \frac{v_t \hat{\tau} s_t}{v_t + \eta}, \tag{15}$$

where s_t is the moving speed of the animal at epoch t. The $g_t(v_t)$ thus characterizes the precision of mobility tracking.

GPS Tracking Trace. We apply the GPS tracking data of a group of bats from [34] to simulate the moving of the object. We set the total number of epochs $T = 60$ with the duration of each epoch as $\hat{\tau} = 30$ seconds. We then compute the average speed s_t of the bat for each epoch t. Fig. 3 gives an illustration of the bat speeds which vary largely in time. The data yields a ratio of maximum and minimum speed $\frac{\max_{t\in[T]} s_t}{\min_{t\in[T]} s_t} = 9.3$ and $\log\theta + 1 = 8.45$ when the rate limit $\beta = 10$.

Energy Arrival Sequence. We consider two types of energy arrival sequence, including the real-world energy harvesting data of a solar panel from [35] and synthesis data following the Poisson process. We scale the overall dataset to simulate different harvesting capabilities of the object. We divide the whole energy harvesting sequence into a number of instances with $T = 60$ epochs. While it is common to assume i.i.d energy harvesting in existing studies [5], [6], we see that the real-world traces follow different patterns at different times, as shown in Fig. 4.

Comparisons of Algorithms. We implement and compare the conceivable alternatives and our algorithm,

$\triangleright$ CR-Reserve: our CR-Reserve algorithm described in Sec. IV-C.

$\triangleright$ CR-Pursuit: the CR-Pursuit$(\ln\theta + 1)$ algorithm described in Sec. IV-B.

$\triangleright$ Greedy: the greedy algorithm that always uses up the current stored energy when the rate limit permits.

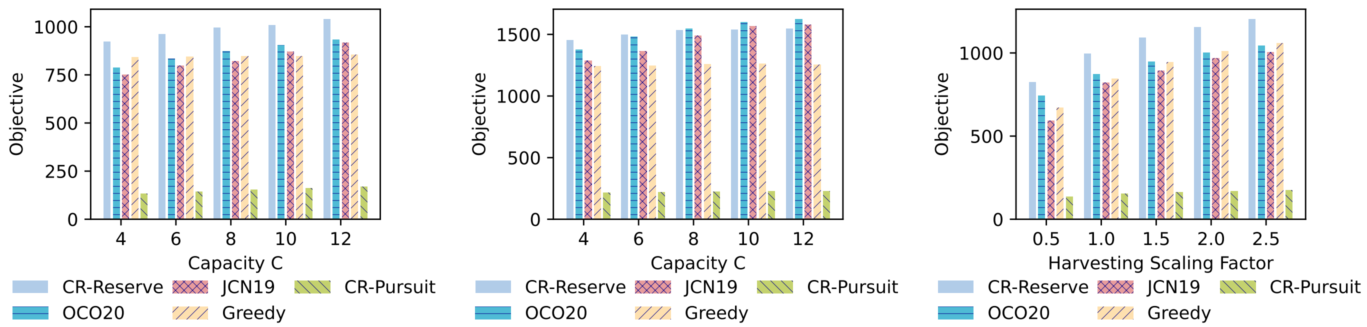
$\triangleright$ OCO20: the online algorithm described in [8]. The algorithm is designed under the online convex optimization (OCO) framework with energy harvesting constraints.

$\triangleright$ JCN19: the threshold policy described in [6]. This policy is optimal to minimize average AoI under Poisson energy arrivals. The same policy is also presented in [5] via a different derivation approach.

B. Performance Comparison with Alternatives

In this subsection, we compare our algorithms with the conceivable alternatives to show the effectiveness of our algorithms. We set the rate limit $\beta = 10$ and vary the capacity and the type of energy harvesting sequence. We run 500 instances for each set of parameters and report their average values. In a default setting, we scale the entire real-world energy harvesting trace to make the average harvested energy one in an epoch.

Performance Comparison. We demonstrate the performance of the algorithms under real-world traces in Fig. 5a. From the results under real-world traces in Fig. 5a, we observe



(a) Performance evaluation under real-world traces of energy harvesting.

(b) Performance evaluation under Poisson energy harvesting.

(c) Performance evaluation under different harvesting capability.

Fig. 5: Performance evaluation under different parameters

TABLE III: Offline-to-online performance ratio for different algorithms and different capacity limits under real-world energy harvesting.

	C=4	C=6	C=8	C=10	C=12
Greedy	3.73	4.57	5.31	5.95	5.77
JCN19	2.04	1.97	2.15	2.01	1.91
OCO20	1.92	2.00	2.05	2.09	2.11
CR-Pursuit	8.45	8.45	8.45	8.45	8.45
CR-Reserve	1.77	1.75	1.74	1.85	1.82
CR-RePursuit	7.16	7.18	7.22	7.27	7.32
CR-RePursuit+	1.68	1.66	1.65	1.64	1.63

that our algorithm CR-Reserve consistently outperforms CR-Pursuit, Greedy, JCN19, and OCO20. In particular, when $C = 10$, we observe that CR-Reserve achieves at least 12% more value compared with alternatives.

We also report the empirical offline-to-online performance ratio in Tab. III. We observe that the empirical performance ratio of our algorithm CR-Reserve is much smaller than the worst-case one, i.e., the competitive ratio, 8.45. Also, it increases slowly with respect to the capacity limit as compared to the other three alternatives. This means that our algorithm consistently achieves a close-to-offline performance under different battery capacities. We observe that the worst-case dedicated algorithm CR-Pursuit performs poorly in practice as it simply maintains the worst-case ratio regardless of the input. With our novel design, CR-Reserve enjoys both the optimal worst-case guarantee and good empirical performance.

Impact of the Energy Arrival Sequence. We also perform simulation under Poisson energy harvesting and show the result in Fig. 5b. We observe that both OCO20 and JCN19 perform well when the energy harvesting sequence satisfies their assumptions. However, their performance degrades significantly when the stochastic assumption fails in real-world data in Fig. 5a. In fact, the performance of OCO20 and JCN19 could be worse than Greedy when the stochastic assumption does not hold. In contrast, our approach CR-Reserve achieves a comparable performance with alternatives under Poisson harvesting while outperforming them notably under real-world energy harvesting sequences. It shows that by considering the problem from the competitive online optimization perspective, our proposed algorithm is more robust against real-world energy harvesting uncertainty.

Impact of Capacity. We investigate the impact of capacity in Fig. 5a and Fig. 5b. We observe that as the capacity increases, the objectives of all algorithms increase. This is attributed to the higher initial energy available and the larger capacity to store harvested energy. We also observe that CR-Reserve achieves a more profound improvement over alternatives at smaller battery capacity, which is the case for small energy harvesting sensors. As capacity increases, however OCO20 and JCN19 have more rapid increments as they tend to utilize the energy more aggressively with the assumption of energy arrivals. Meanwhile, CR-Reserve, which does not assume any energy harvesting information, maintains a proportional utilization of energy, resulting in a slower rate of increase. Greedy, which promptly allocates the increasing energy, experiences a limited performance increment due to the concavity of the value functions.

Impact of Harvesting Capability. We then study the impact of the harvesting capability of sensors. In particular, we scale the real-world energy harvesting trace such that the average harvested energy equals to the scaling factor $\gamma = 0.5, ..., 2.5$. We set the capacity $C = 8$. We present the results in Fig. 5c. We observe that as the scaling factor γ increases, all algorithms benefit from additional energy harvesting and demonstrate better performance. Meanwhile, the increment tends to diminish with increasing γ. We believe it is due to the capacity limit that even with higher harvesting capability, there may not be enough capacity to store. In addition, CR-Reserve consistently outperforms all alternatives.

VI. Conclusion

In this paper, we study the AoI optimization problem of an energy harvesting system. Our goal is to achieve the maximum time-varying values of maintaining a low AoI by optimizing the energy allocation. We consider a competitive online approach without relying on stochastic information about the energy harvesting process or future information about the time-varying value functions. We develop an online algorithm CR-Reserve that achieves the optimal competitive ratio $\ln\theta + 1$ among all deterministic and randomized online algorithms. Notably, our algorithm CR-Reserve further exploits the non-worst case input and achieves a significantly improved empirical performance against the worst-case dedicated solution CR-Pursuit. We also compare with alternatives including the state-of-the-art approaches that rely on the i.i.d assumption of the energy harvesting process. We show that under real-world traces, our approach consistently improves the performance against existing algorithms in various settings.

As for future work, it is interesting to study the multi-source case where multiple sources send their updates to a common receiver through a shared channel. Another interesting direction is to apply our algorithm and ideas to broader scenarios, including the applications of AoI for different objectives.

APPENDIX

We first show that for a special case of the problem, GAoI without the battery capacity limit, named GAoI_∞, CR-Pursuit achieves the optimal competitive ratio $\ln\theta+1$. In such a case, we have $h_t = \hat{h}_t, \forall t$ and no energy harvesting constraints (3).

Lemma 3. *CR-Pursuit($\ln\theta+1$) is ($\ln\theta+1$)-competitive for GAoI_∞.*

Due to the space limit, we only discuss the proof idea here. We show that by a carefully-designed decomposition of the value functions, we can apply the divide-and-conquer approach to divide GAoI_∞ into subproblems without energy harvesting, and the initial energy of the subproblems is either the capacity C or the energy arrivals h_t. For each subproblem, CR-Pursuit(π) can satisfy the individual energy allocation constraint with $\pi = \ln\theta + 1$ [10]. Further, we show that the output of CR-Pursuit($\ln\theta+1$) is upper bounded by the joint output of the subproblems, and thus feasible for GAoI_∞.

We then show two lemmas on the energy harvesting of the offline optimal solutions for GAoI. Let the optimal energy allocation under input up to t as $v^t_\tau, \forall\tau\in[t]$. Let the optimal energy harvesting under input up to t as $h^t_\tau, \forall\tau\in[t]$. We have the following relationship between v^t_τ and $h^t_\tau, \forall\tau\in[t]$.

Lemma 4. *There exists an optimal solution to the offline optimal under input up to epoch t, say $\{v^t_\tau, h^t_\tau\}_{\tau\in[t]}$ such that,*

$$h^t_\tau = \min\left\{\hat{h}_\tau, \sum_{s=1}^{\tau} v^t_s - \sum_{s=1}^{\tau-1} h^t_s\right\}. \tag{16}$$

We consider the optimal solutions $\{v^t_\tau, h^t_\tau\}_{\tau\in[t]}$ satisfying (16). It leads to the following property of such an optimal energy harvesting solution among different epochs.

Lemma 5. *We have*

$$h^t_\tau = h^s_\tau, \forall s, t \geq \tau, \forall\tau\in[T]. \tag{17}$$

With the above lemma, we can consider a fixed offline optimal energy harvesting sequence at all epochs, which we denote as $h^*_t, \forall t$.

Proof of Theorem 1. As discussed in Sec. IV-A, it is sufficient to show that CR-Pursuit(π) always satisfies the energy allocation constraints (2).

Suppose that at all epochs, there is sufficient capacity for CR-Pursuit($\ln\theta+1$), i.e., $\tilde{h}_t = \hat{h}_t, \forall t \in [T]$. It is not hard to show that the allocation of CR-Pursuit($\ln\theta+1$) is upper bounded by that under the case with no capacity limit (observing that the offline optimal is further constrained by the capacity limit). And according to Lemma 3, we can conclude that CR-Pursuit(π) with $\pi = \ln\theta+1$ is feasible under such an input.

Suppose the condition fails. Let us consider the first time that the condition does not hold, say $\tilde{t}$. In such a case, we have that at the end of epoch $\tilde{t}$, the stored energy of the online algorithm equals the battery capacity or the initial energy C. At epoch $\tilde{t}$, we have that $\sum_{\tau=1}^{\tilde{t}} h^*_\tau \leq \sum_{\tau=1}^{\tilde{t}} v^{\tilde{t}}_\tau$. We can decompose the offline optimal solution given input up to epoch $\tilde{t}$ into two parts. Let the optimal solution be $v^{\tilde{t}}_\tau, \forall\tau\in[\tilde{t}]$. We define the first part as those utilize all energy harvesting up to epoch $\tilde{t}$, which is GAoI in epoch $[1,\tilde{t}]$ with an additional constraint at epoch $\tilde{t}$,

$$\sum_{\tau=1}^{\tilde{t}} v_\tau = \sum_{\tau=1}^{\tilde{t}} h^*_\tau. \tag{18}$$

And we denote the problem as GAoI^1_t. Let the optimal solution to GAoI^1_t as $\{v^{\tilde{t}}_{\tau,1}\}_{\tau\in[\tilde{t}]}$. And, we let the remaining allocated energy as the second part, i.e., $v^{\tilde{t}}_{\tau,2} = v^{\tilde{t}}_\tau - v^{\tilde{t}}_{\tau,1}, \forall\tau\in[\tilde{t}]$. We can show that after $\tilde{t}$, the optimal solution would only decrease the second part to leave more energy to be allocated in the future epochs. The first part allocating the same amount of harvested energy would not change due to constraint (3). So, for all epochs after $\tilde{t}$, we can fix the first part and reoptimize the second part with the newly revealed input. That is, GAoI^2_t

$$\max \quad \sum_{\tau=1}^{t}\left(g_\tau(v_\tau + v^{\tilde{t}}_{\tau,1}) - g_t(v^{\tilde{t}}_{\tau,1})\right) + \sum_{\tau=\tilde{t}+1}^{t} g_\tau(v_\tau) \tag{19}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{s} v_\tau \leq C + \sum_{\tau=\tilde{t}+1}^{s} h^*_\tau, \forall s\in[t] \tag{20}$$

$$\sum_{\tau=1}^{s} v_\tau - \sum_{\tau=\tilde{t}+1}^{s} h^*_\tau \geq 0, \forall s\in[t] \tag{21}$$

$$\text{var.} \quad 0 \leq v_\tau \leq v^{\tilde{t}}_{\tau,2}, \forall\tau\in[\tilde{t}], \tag{22}$$

$$0 \leq v_\tau \leq \beta, \forall\tau\in[\tilde{t}+1, t]. \tag{23}$$

We can show that the combination of these two parts remains the optimal solution for any epoch after $\tilde{t}$.

Then, starting from $\tilde{t}$, the online algorithm is equivalent to pursue the performance ratio $\ln\theta+1$ against the optimal solution to GAoI^2_t, as the first part is fixed, and the increment in the offline optimal only comes from the second part. We note that in the second part, there is no energy harvesting before epoch $\tilde{t}$, we can keep using these arguments at all epochs that the online algorithm does not harvest all the energy arrival and returns to the full capacity C. Finally, we come to a case that the second part of the problem with no energy harvesting before an epoch, and the online algorithm harvests all the rest energy arrival. In such a case, the output of the online algorithm is bounded by that running under the input of the second part problem from the first epoch and without a capacity limit. And it thus remains feasible. □

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