NEW CONGESTION CONTROL SCHEMES OVER WIRELESS NETWORKS: STABILITY ANALYSIS

Minghua Chen * **Alessandro Abate** *
Shankar Sastry *

* *Department of Electrical Engineering and Computer Science*
University of California at Berkeley
Berkeley, CA 94720
{*minghua, aabate, sastry*}*@eecs.berkeley.edu*

Abstract: The objective of this work is to introduce two original flow control schemes for wireless networks. The mathematical underpinnings lie on the recently-developed congestion control models for Transmission-Control-Protocol(TCP)-like schemes; more precisely, the model proposed by Kelly for the wired case is taken as a template, and properly extended to the more involved wireless setting. We introduce two ways to modify a part of the model; the first is through a static law, and the second via a dynamic one. In both cases, we prove the global stability of the schemes, and present a convergence rate study and a stochastic analysis.

Keywords: Communication networks and protocols, flow control, large-scale systems, stability robustness.

1. INTRODUCTION

Congestion Schemes for Communication Networks have proven to be of the utmost importance when applied to key applications such as the Internet; for instance, the TCP protocol is widely regarded as the most known and employed scheme for the exchange of digital information (Jacobson, 1998).

A network is described via two of its entities, the users and the links. TCP (here we shall focus on the Reno case) regulates the packets sent by the users in the network, in order to avoid channel congestions. This is done by increasing the window size, i.e. the number of packets sent per unit of time, when no packet is lost during the previous round trip time, and halving it otherwise; therefore, it assumes that lost packets are symptomatic of congestion. The number of packets lost is described via loss functions at the links. The update mechanisms for both rates and losses are distributed, i.e. based on local information only.

The analysis of this and other similar protocols have focused on many issues: modeling (Kelly, 2003)-(Kelly *et al.*, Dec 1999)-(Alpcan and Basar, Dec 2003)-(Kunniyur and Srikant, Oct 2003), stability (Kunniyur and Srikant, Oct 2003)-(Johari and Tan, Dec 2001)-(Paganini *et al.*, to appear)-(Wang and Paganini, Dec 2003), robustness to delays (Vinnicombe, 2002), assessment of the utility

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functions for the underlying optimization problem (Low and Lapsley, 1999), and its dual interpretation (Low, Aug 2003). All these efforts have been focused on the wired case.

Congestion Control over Wireless Networks poses additional challenges; in this extended framework, packet loss is due not only to congestion at the link, but also to physical channel error. As a consequence, in practice the network could be under utilized. A recent work on MULTFRC, (Chen and Zakhor, 2004), proposes an original dynamic scheme to improve the performance over wireless network. In a paper by the same authors, (Chen *et al.*, 2005), a static scheme and a dynamic one are proposed; the global stability of both schemes are proved in generality, and the study of delay sensitivity of the first one is also tackled. This work aims at completing the former paper, and will be presented as follows. First, the mathematical framework will be introduced and the two models proposed. We shall then quote the stability results for both schemes. Then, the rate of convergence of the two schemes will be derived, and a stochastic analysis will be developed. Some final considerations on the proposed scheme and the discussion of future work will close up the paper.

In a companion paper by the same authors, (Abate *et al.*, 2005), the delay sensitivity of the static and the dynamic case is analyzed. Sufficient structural conditions on the dynamic scheme are introduced to ensure local stability with respect to the delays in the system; the oscillations of the solutions are also qualitatively analyzed.

2. PROBLEM SETUP

2.1 The model for the wired case

A network is described via its J *resources*, its links, and its R *users*, i.e. sender-receiver pairs, which can also be conceived as subsets of J, the *routes*. Each link j has a finite capacity C_j. The connections of the network are described via a matrix $A = (a_{jr}, j \in J, r \in R)$, where $a_{jr} = 1$ if $j \in r$. Every user is endowed with a sending rate $x_r \geq 0$ and a utility function $U_r(x_r)$, assumed to be increasingly, strictly concave and $\mathcal{C}^1$. Kelly was the first to interpret the flow control as the solution of the following concave maximization problem, dependent on the aggregate utility functions for the rates and on some costs on the links (Kelly *et al.*, Dec 1999):

$$\max \sum_{r \in R} U_r(x_r) - \sum_{j \in J} P_j\big(\sum_{s: j \in s} x_s\big), \tag{1}$$

where the cost functions $P_j(\cdot)$ are defined as:

$$P_j(y) = \int_0^y p_j(z)\, dz. \tag{2}$$

Here $p_j(y)$ is the price at the link j, and is assumed to be non-negative, continuous and increasing function; moreover, it is expected to depend on the aggregate rate passing through the link. Throughout this paper we use the following form for $p_j(y)$, which can be interpreted as the "packet loss rate",

$$p_j(y) = \frac{(y - C_j)^+}{y}. \tag{3}$$

The end-to-end packet loss rate for user r is then $1 - \prod_{j \in r}[1 - p_j(\sum_{s:j \in s} x_s)]$, which is approximately equal to $\sum_{j \in r} p_j(\sum_{s:j \in s} x_s)$ assuming $p_j(\sum_{s:j \in s} x_s)$ is small. We consider the following scheme, which is the continuous-time version of TCP-like additive increase, multiplicative decrease algorithm:

$$\frac{d}{dt} x_r(t) = k_r \big[w_r^o - x_r(t) \sum_{j \in r} \mu_j(t)\big], r \in R \tag{4}$$

with k_r being a positive scale factor affecting the adaptation rate, and w_r^o a weight that can be interpreted as pay per unit time; the congestion signal is generated at a link j as follows:

$$\mu_j(t) = p_j\big(\sum_{s:j \in s} x_s(t)\big). \tag{5}$$

With this primal scheme (4)-(5), the unique, globally and asymptotically stable point of the network, denoted by $x^o = (x_r^o, r \in R)$ [2], is given by

$$x_r^o = \frac{w_r^o}{\sum_{j \in r} p_j\left(\sum_{s:j \in s} x_s^o\right)}, \quad r \in R; \tag{6}$$

This unique solution is also the optimal solution for the optimization problem in (1). The solution is desirable in the sense that the network's bottlenecks are fully utilized, the total net utility is maximized, and the users are proportionally fair to each other (Kelly *et al.*, Dec 1999).

2.2 The wireless case

One of the main differences between the wired case and the wireless one is the presence of physical channel errors in this latter case; in the setting of our model, these affect the packet loss rate, which in the wired case depends only on the congestion measure. Assume every link j is affected by the

[2] In order to keep the notation light, throughout the entire paper user or link variables with no subscript directly denote vectorial quantities.

wireless packet error rate ϵ_j, the new price function ν_j is:

$$\nu_j(t) = p_j\Big(\sum_{s:j\in s} x_s(t)\Big) + \Big(1 - p_j\Big(\sum_{s:j\in s} x_s(t)\Big)\Big)\epsilon_j$$
$$\triangleq q_j\Big(\sum_{s:j\in s} x_s(t)\Big) \geq p_j\Big(\sum_{s:j\in s} x_s(t)\Big). \qquad (7)$$

The utility maximization problem for wireless network still has the form in (1), as none of the utility or the cost is a function of $\epsilon_j, j \in J$. The primal scheme (4) will adapt itself according to this new price functions q_j, which have the same structural properties as the old p_j; the equilibrium point of the system will therefore change accordingly. From an optimization prospective, the new equilibrium is a suboptimal solution for the optimization problem in (1). Exploiting duality arguments, this also means that some of the bottleneck links may be under utilized.

The task of this work is to address and solve this underutilization problem. Unlike many existing approaches, which try to provide the user with the feedback of the exact price $\sum_{j\in r} \mu_j(t)$, or with an estimate of it, our approach is to act on the term w_r, which physically corresponds to adjusting the number of connections the user has to the network (Chen and Zakhor, 2004) (Chen *et al.*, 2005). In the first approach, w_r is instantaneously adjusted using a static function with respect to $\nu_j(t)$ and $\mu_j(t)$; in the second approach, w_r is gradually adjusted by a dynamic update law. Both of these are end-to-end application layer based schemes, since changing w_r can be implemented by changing the number of connections opened by one user. Therefore they require no modification to either the network infrastructure, e.g. router, or to the network protocols, e.g. TCP.

3. TWO NEW CONTROL SCHEMES

3.1 Static Update

Assume the term ω_r is time dependent, $w_r(t)$, and is adjusted according to the following law:

$$w_r(t) = w_r^o \frac{\sum_{j\in r} \nu_j(t)}{\sum_{j\in r} \mu_j(t)}. \qquad (8)$$

Then, the source rate for user r then is given by:

$$\frac{d}{dt} x_r(t) = k_r\big[w_r(t) - x_r(t)\sum_{j\in r} \nu_j(t)\big]. \qquad (9)$$

A straightforward analysis shows how, under this change, the equilibrium of this system is again x^o. Intuitively, seen from (8), if the noise is large, i.e. $\nu_j(t) > \mu_j(t)$, it can be counterbalanced by an increase in $w_r(t)$.

3.2 Dynamic Update

Rather than using an instantaneous adaptation rule, we can propose a dynamic update for w_r:

$$\frac{d}{dt} w_r(t) = c_r\Big[w_r^o - w_r(t)\frac{\sum_{j\in r} p_j(\sum_{s:j\in s} x_s(t))}{\sum_{j\in r} q_j(\sum_{s:j\in s} x_s(t))}\Big]. \quad (10)$$

The equilibrium points of the new, extended system are composed of a first part given by the vector x^o and a second part, for the new dynamics, given by $w_r^o \sum_{j\in r} \nu_j(t)/\sum_{j\in r} \mu_j(t)$. The system of coupled equations (4)-(5)-(10) is strongly nonlinear and asymmetric.

4. GLOBAL STABILITY

Stability is the first requirement one needs to investigate on a dynamical system. In our problem it also has the property that the rates allocation among users at equilibrium are fair, all network bottlenecks are fully utilized, avoiding at the same time congestion collapse (Section I in (Jacobson, 1998)). We state here two theorems, the proofs of which can be found in (Chen *et al.*, 2005); simulations to verify these theorems are also discussed there.

4.1 Static Update

Theorem 1. System (4)-(5)-(8) is globally asymptotically stable with the following Lyapunov function:

$$V(x) = \sum_{r\in R} w_r^o \log x_r - \sum_{j\in J} \int_0^{\sum_{s:j\in s} x_s} p_j(y)dy. \qquad (11)$$

All trajectories converge to the equilibrium point x^o in (6) that maximizes $V(x)$.

4.2 Dynamic Update

The key assumption that we make in this section is that *the dynamics corresponding to (4)-(5) and (10) evolve in two different time scales; the first in a faster one, while the second in a slower one.* These two relations can be vectorized as [3]:

$$\varepsilon \dot{x}(t) = kw(t) - diag(kx(t))A^T(p \circ Ax(t)) \triangleq F(x(t)); \qquad (12)$$

[3] The vectors kw and kx are built through the componentwise products of the vectors k, w and k, x respectively.

$$\dot{w}(t) \triangleq G(w(t), x(t)). \tag{13}$$

The composition operation on p is intended to be performed componentwise. In this case [4], the following holds:

Theorem 2. For the overall system (12)-(13), featuring prices that are functions of only the aggregate rates, e.g. (3), if the following is true:

- Equation (12) has a region of equilibrium points $F(x) = 0$ that identifies a manifold $w = h(x)$;
- Within this manifold, Equation (13), also known as the *reduced system*, has a unique global equilibrium which is the solution of $G(h(x), x) = 0$;
- The functions F, G, h and their partial derivatives are bounded near the global equilibrium;
- The equilibrium manifold for the *boundary-layer system* [5] is exponentially stable;
- The global equilibrium of the reduced system $\dot{w} = \frac{\partial h}{\partial x}\dot{x} = G(x, h(x))$ is asymptotically stable;

then, there exists an ε^* such that, $\forall \varepsilon \leq \varepsilon^*$, the equilibrium point of the composite system is asymptotically stable [6].

5. RATE OF CONVERGENCE AND STOCHASTIC ANALYSIS

Although global stability implies all the trajectories converges to the unique equilibrium, it does not indicate how fast they do so and upon what this depends. The analysis on the rate of convergence gives insights to the latter questions and hints at designing improved protocols. The real implementation of the proposed schemes depends on accurate measure on the packet loss rates. However, the real network environment always introduces noise to the measurements. By modeling the effects of all these disturbances as Brownian motion perturbations on the deterministic system, an analysis can help to understand robustness of the schemes to these inevitable disturbances.

5.1 Rate of convergence for the Static Update

To carry out the rate of convergence analysis, let $l_r = k_r \sum_{j \in r} q_j / \sum_{j \in r} p_j$ and $x_r(t) = x_r^o + (l_r x_r^o)^{1/2} y_r(t)$; we linearize the static system (9) around the equilibrium point $x^o = [x_r^o, r \in R]$:

$$\dot{y_r}(t) = -\big(l_r y_r(t) \sum_{j \in r} p_j + (l_r x_r^o)^{1/2} \sum_{j \in r} p'_j \sum_{s: j \in s} y_s(t)\,(l_s x_s^o)^{1/2}\big). \tag{14}$$

We may write it in matrix form as:

$$\dot{Y}(t) = -\Gamma^T \Phi \Gamma\, Y(t), \tag{15}$$

where Γ is an orthogonal matrix, $\Gamma^T\Gamma = I$, and $\Phi = diag(\phi_r, r \in R)$ is the matrix of eigenvalues, necessarily positive, of the following real, symmetric, positive definite matrix

$$\Gamma^T \Phi \Gamma = diag\{\sum_{j \in r} p_j\} L + L^{1/2} X^{1/2} A^T P' A X^{1/2} L^{1/2}, \tag{16}$$

where $X = diag\{x_r^o, r \in R\}$, $P' = diag\{p'_j, j \in J\}$, $L = diag\{l_r\}$.

Hence the rate of convergence to the equilibrium point is determined by the smallest eigenvalue, $\phi_r, r \in R$, of the matrix (16).

5.2 Stochastic analysis for the Static Update

Here we consider the stochastic perturbation of the linearized equation (15). The perturbations can be caused by the random nature of packet loss. Let

$$dY(t) = -\Gamma^T \Phi \Gamma\, Y(t) dt - F dB(t), \tag{17}$$

where F is an arbitrary $R \times I$ matrix and $B(t) = (B_i(t), i \in I)$ is a collection of independent standard Brownian motions, extended to $-\infty < t < \infty$. Following the similar procedure for the stochastic analysis (Kelly *et al.*, Dec 1999), we conclude that the stationary solution to the system (17) has a multivariate normal distribution, $Y(t) \sim N(0, \Sigma)$, where

$$\Sigma = E[Y(t)Y(t)^T] = \Gamma^T [\Gamma F; \Phi] \Gamma,$$

where the symmetric matrix $[\Gamma F; \Phi]$ is given by $[\Gamma F; \Phi]_{rs} = \left[\int_{-\infty}^{0} e^{\tau\Phi} \Gamma F F^T \Gamma^T e^{\tau\Phi}\right]_{rs} = [\Gamma F F^T \Gamma^T]_{rs} / (\phi_r + \phi_s)$.

5.3 Rate of convergence for the Dynamic Update

Let $x_r(t) = x_r^o + (x_r^o)^{1/2} y_r(t)$, and $w_r(t) = w_r + (x_r^o)^{1/2} z_r(t)$. Linearizing the system (4)-(5)-(10) around equilibrium point (x^o, w) results in

[4] For simplicity, we shall consider the simplified case $c = 1$.
[5] This system is obtained through a rescaling of time, $\tau = t/\epsilon$ and letting $\epsilon \to 0$.
[6] Refer to literature on stability for singularly-perturbed, non-linear systems, like (Sastry, 1999).

$$\dot{y_r}(t) = k_r\big(z_r(t) - y_r(t)\sum_{j\in r} q_j - (x_r^o)^{1/2}\sum_{j\in r} q_j' \sum_{s:j\in s} y_s(t)(x_s^o)^{1/2}\big), \quad (18)$$

and

$$\begin{aligned}\dot{z_r}(t) = &-c_r\big(z_r(t)\frac{\sum_{j\in r} p_j}{\sum_{j\in r} q_j} \\ &+ (x_r^o)^{1/2}\sum_{j\in r} p_j' \sum_{s:j\in s} y_s(t)(x_s^o)^{1/2} \quad (19)\\ &- (x_r^o)^{1/2}\frac{\sum_{j\in r} p_j}{\sum_{j\in r} q_j}\sum_{j\in r} q_j' \sum_{s:j\in s} y_s(t)(x_s^o)^{1/2}\big).\end{aligned}$$

This is a coupled, multivariate linear system; we apply two timescale decomposition to decouple the system into boundary system and reduced system to carry out analysis under the classical singular perturbation framework, as we did in the global asymptotic convergence analysis.

The boundary system is described by (18) with $z_r(t)$ to be fixed. We may write boundary system in matrix form as

$$\dot{Y}(t) = -KB\,Y(t) + KZ, \quad (20)$$

where

$$B = diag\{\sum_{j\in r} q_j\} + X^{1/2}A^T Q' A X^{1/2}$$

is a symmetric, positive definite matrix, $Q' = diag\{q_j', j \in J\}$, and $K = diag\{k_r, r \in R\}$. The product KB is a diagonalizable matrix with all eigenvalues to be positive (Horn and Johnson, 1985); hence, the linearized system (20) converges to a manifold, i.e. its equilibrium $Y(t) = B^{-1}Z$, with the rate of convergence determined by the smallest eigenvalue of matrix KB.

On the larger timescale, we analyze the rate of convergence for the reduced system, described by (19) on the manifold $Y(t) = B^{-1}Z(t)$. We may write the reduced system in matrix form as

$$\dot{Z}(t) = -C\,D\,Y(t) = -C\,D\,B^{-1}Z(t), \quad (21)$$

where $C = diag\{c_r, r \in R\}$ and

$$D = diag\{\sum_{j\in r} p_j\} + X^{1/2}A^T P' A X^{1/2},$$

Hence, in the case where $c_r = c, r \in R$ [7], the linearized reduced system (21) converges to the equilibrium point (x^o, w), with the rate of convergence determined by the smallest eigenvalue of matrix DB^{-1}.

In summary, the rate of convergence of the entire system depends on how it converges on large timescale, i.e. of the reduced system, and hence is determined by the smallest eigenvalue of the matrix DB^{-1}.

5.4 Stochastic analysis for the Dynamic Update

The effects of stochastic disturbance on the dynamic update scheme, modeled as a singular perturbation nonlinear system, is more subtle than the static update case. According to (Sastry, 1983), the correct procedure is to first get the stationary distribution of x in boundary system; then investigate behavior of reduced system based on an modified version of nonlinear differential equations that is averaged over the stationary distribution of x. The analysis can be hard. However, as we only investigate the linearization system around the equilibrium, it turns out the averaged linear differential equations are exactly the *same* as the original ones.

Therefore, to analyze the effects of stochastic disturbance on the dynamic update scheme locally, is equivalent to investigate the effects on both boundary system and reduced system using a standard procedure similar to the one in Section 5.2, i.e. the boundary system is investigated in the way as if x is converged to the equilibrium manifold rather than the stationary distribution.

5.4.1. Boundary layer system Here we first consider the stochastic perturbations of the linearized equation (20) as follows:

$$dY(t) = -KB\,Y(t) + KZ - F_1 dG(t), \quad (22)$$

where F_1 is an arbitrary $R \times I$ matrix and $G(t) = (G_i(t), i \in I)$ is a collection of independent standard Brownian motions, extended to $-\infty < t < \infty$. Following the similar procedure of stochastic analysis part in Section 5.2, we conclude that in the stationary stage, $Y(t) \sim N(B^{-1}Z, \Sigma_1)$. Let the diagonalizable matrix $B\,K = S_1^{-1}\Phi S_1$, then

$$\Sigma_1 = KS_1^{-1}[S_1F_1;\Phi]S_1K. \quad (23)$$

where matrix $[S_1F_1;\Phi]$ is given by $[S_1F_1;\Phi]_{rs} = \frac{[S_1F_1F_1^TS_1^{-1}]_{rs}}{\phi_r+\phi_s}$.

5.4.2. Reduced system Next we start to consider the stochastic perturbations of the linearized

[7] In the general case with different values for $c_r, r \in R$, we expect the eigenvalues of the product CDB^{-1} to be all positive to ensure the system is stable. However, at this moment, this is nothing more than a conjecture.

equation (21), under the assumption $c_r = c, r \in R$, as follows:

$$dZ(t) = -c\,D\,B^{-1}Z(t) - F_2 dE(t), \qquad (24)$$

where F_2 is an arbitrary $R \times I$ matrix and $E(t) = (E_i(t), i \in I)$ is a collection of independent standard Brownian motions, extended to $-\infty < t < \infty$. Following the similar procedure of stochastic analysis part in Section 5.2, $Z(t)$ again has stationary distribution $N(0, \Sigma_2)$. Let the diagonalizable matrix $DB^{-1} = S_2^{-1}\Phi S_2$, then

$$\Sigma_2 = S_2^{-1}[S_2F_2; \Phi]S_2, \qquad (25)$$

where matrix $[S_2F_2; \Phi]$ is given by $[S_2F_2; \Phi]_{rs} = \frac{[S_2F_2F_2^TS_2^{-1}]_{rs}}{\phi_r+\phi_s}$.

In summary, stochastic perturbations introduce additive multivariate normal distributions with zero mean and variances given in (23) and (25) to the equilibrium solutions in boundary system and reduced system. Since both systems are robust to stochastic perturbations around the equilibrium point (x^o, w), the entire system is robust to stochastic perturbations around (x^o, w) according to the arguments at the beginning of this section.

6. CONCLUSIONS

In this paper we proposed two new flow control schemes over wireless networks, a static and a dynamic one. We continued the analysis of the structural properties of the schemes started in (Chen *et al.*, 2005), and focused on the stochastic study of stability, and on the computation of the rate of convergence of the proposed schemes. The study of delay sensitivity and of the quality of the oscillations due to delays in the system has been investigated in (Abate *et al.*, 2005); this second work also contains simulations. This complete study advocated the applicability of the algorithms in real wireless networks.

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