

Towards Real-Time Energy Generation Scheduling in Microgrids with Performance Guarantee

Lian Lu*, Jinlong Tu*, Chi-Kin Chau†, Minghua Chen*, Zhao Xu‡ and Xiaojun Lin§

*The Chinese University of Hong Kong, Hong Kong, †Masdar Institute, UAE,

‡The Hong Kong Polytechnic University, Hong Kong, §Purdue University, the U.S.

Abstract—Microgrids are local electricity systems that integrate demand and distributed energy resources. They represent a basic building block for future electricity systems that are more cost-effective than conventional grids. Real-time energy generation scheduling is an important issue in microgrid operations. However, common approaches in conventional grids suffer from various limitations and do not apply to microgrids: (1) stochastic optimization approaches rely on stochastic modeling, which is difficult to obtain in microgrids with significant wind penetration and varying demand responses participation; (2) robust optimization approaches provide no cost-efficiency performance guarantee. In this paper, we present a real-time scheduling solution without stochastic modeling, considering a flexible time-window of prediction and providing performance guarantee with minimal risk. Our solution is an generalization of our previous work from homogeneous local generation to heterogeneous settings. We evaluate the performance of our proposed solution using the operating trace of a pilot microgrid on the Bornholm Island, Denmark. We show that our solution achieves a performance close to perfect dispatch, and is robust to prediction errors.

I. INTRODUCTION

Recently, there has been an increased penetration of distributed renewable energy sources, demand responses participation and coupled resources (*e.g.*, co-generation), particularly in emerging microgrid systems. This creates enormous challenges to the design of reliable but yet cost-effective generation scheduling strategies that can balance time-varying demand and supply at roughly the same time.

Traditionally, generation scheduling problem has been extensively studied based on Unit Commitment (UC) [15] and Economic Dispatch (ED) [8] problems. Unfortunately, the classical strategies cannot cope well with the rapidly varying intermittent sources (*e.g.*, wind power) and demand responses. In particular, if we consider microgrids, the abrupt changes in local weather condition may have a dramatic impact that cannot be amortized as in the larger-scale national grid. For instance, in Fig. 4 we examine one-week traces of electricity demand and wind power output of Bornholm Island, Denmark. We observe that the net electricity demand inherits a large degree of variability from the wind generation, casting a challenge for accurate prediction. Furthermore, coupled energy resources (*e.g.*, co-generation) complicate the scheduling decisions. For instance, in Fig. 4 the heat demand exhibits a different stochastic pattern, complicating the prediction of overall energy demand.

To reduce the impact from prediction errors, *real-time* scheduling has been increasingly advocated in the community

[14], which requires commitment and dispatch decisions as frequently as in hourly basis. To realize real-time scheduling, several solutions have been proposed. Stochastic optimization approach [16] is one of the popular solutions, which however suffers from inaccurate a-priori assumptions and parameters of stochastic modeling. Another approach is robust optimization [17], which optimizes commitment and dispatch decisions with respect to a large set of demand possibilities, under security constraints. But robust optimization cannot provide a cost-efficiency guarantee against perfect prediction result.

In our previous work in [13], we present a scheduling algorithm called CHASE (Competitive Heuristic Algorithm for Scheduling Energy-generation). CHASE does not rely on stochastic modeling and yet provides a guaranteed cost-efficiency against perfect prediction. Furthermore, we can theoretically show that our scheduling algorithm CHASE is the optimal one with respect to the competitive ratio, with a mild condition¹. The implication is that CHASE can ensure energy generation with *minimal risk*. Our previous study in [13] focuses on a homogeneous setting where local co-generation generators have the same parameters such as maximum output levels, minimum on/off times and ramping-up/down rate limits. In practice, however, microgrids may employ different types of co-generation generators with heterogeneous operating constraints. In this paper, we extend CHASE to the general setting where local generators can have heterogeneous parameters, and thus extend the applicability of CHASE beyond the homogenous scenarios studied in [13].

Conceptually, our proposed scheduling approach can be implemented in existing microgrids as described in Fig. 1.

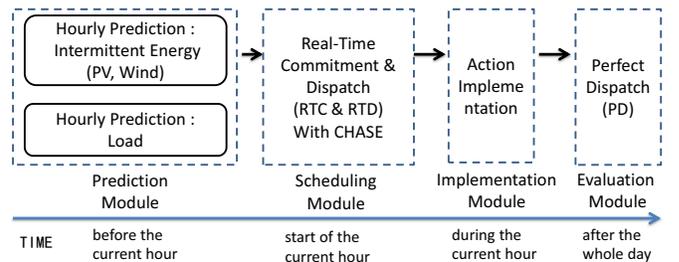


Fig. 1: Proposed implementation of CHASE in microgrids

¹The competitive ratio is based on online competitive analysis [5], and here refers to the cost of scheduling algorithm without prediction over the optimal cost with perfect prediction over all cases.

There are four modules in the implementation. First, we employ the prediction module to obtain the intermittent energy and load of the next hour. We note that the short time prediction is much more precise than the day-ahead prediction. Based on hourly prediction, we carry the real-time scheduling with CHASE. Next, the schedule is executed. After a day of commitment and dispatch execution, the performance is evaluated by *Perfect Dispatch* (PD). The notion of PD is proposed by *PJM*, which refers to the generation scheduling with perfect prediction. Although this solution is hypothetical, a PD solution serves as a baseline for benchmarking actual daily grid performance [9].

Our study provides the following contributions:

- In Sec. II, we formulate the microgrid energy generation scheduling problem for a general scenario where microgrids employ intermittent renewable energy sources and diverse co-generation generators with *heterogeneous* operating constraints including maximum output levels and ramping-up/-down rate limits. This generalizes the homogeneous setting considered in our previous work [13] where local generators have identical parameters. We then generalize our recently-proposed algorithm CHASE to the new heterogeneous scenarios considered in this paper in Sec. III.
- We also show in Sec. III that for *arbitrarily uncertain* demand and intermittent renewable energy generation, our extended algorithm CHASE achieves a cost within a constant times of that achieved by PD. In other words, CHASE has a bounded operational risk under any uncertain circumstances. This is in contrast to conventional generation scheduling approaches which might have unbounded operational risk, *i.e.*, their costs can be arbitrarily larger than that achieved by PD, if their assumed models do not match the input uncertainty in practice.
- We evaluate our algorithm using the operating trace of a pilot microgrid on the Bornholm Island, Denmark in Sec. IV. We show that our algorithm achieves a performance close to PD, and is robust to prediction errors.

II. PROBLEM SETTINGS

We consider a typical scenario where a microgrid orchestrates different energy generation sources to minimize cost for satisfying both local electricity and head demands simultaneously, while meeting operational constraints of electricity system. We will formulate a microgrid cost minimization problem (MCMP) that incorporates intermittent energy demands, time-varying electricity prices, local generation capabilities and co-generation in Sec. II-B.

We define the notations in Table I.

A. Model

Intermittent Energy Demands: We consider arbitrary renewable energy supply (*e.g.*, wind). Let the net demand (*i.e.*, the residual electricity demand not balanced by wind generation) at time t be $a(t)$. Note that we do not rely on any specific stochastic model of $a(t)$.

External Power from Electricity Grid: The microgrid can obtain external electricity supply from the central grid for

T	The total number of intervals (unit: hour)
N	The total number of local generators
n	The id of the n -th local generator, $1 \leq n \leq N$
β	The startup cost of local generator (\$)
c_m	The sunk cost per interval of running local generator (\$)
c_o	The incremental operational cost per interval of running local generator to output an additional unit of power (\$/Watt)
c_g	The price per unit of heat obtained externally using natural gas (\$/Watt)
L_n	The maximum power output of the n -th generator (Watt), $1 \leq n \leq N$.
T_{on}^n	The minimum on-time of the n -th generator, once it is turned on
T_{off}^n	The minimum off-time of the n -th generator, once it is turned off
R_{up}^n	The maximum ramping-up rate of the n -th generator (Watt/hour)
R_{dw}^n	The maximum ramping-down rate of the n -th generator (Watt/hour)
η	The heat recovery efficiency of co-generation
$a(t)$	The net power demand minus the instantaneous wind power supply and stored power from battery (Watt)
$h(t)$	The space heating demand (Watt)
$p(t)$	The spot price per unit of power obtained from the electricity grid ($P_{\min} \leq p(t) \leq P_{\max}$) (\$/Watt)
$\sigma(t)$	The joint input at time t : $\sigma(t) \triangleq (a(t), h(t), p(t))$
$y_n(t)$	The on/off status of the n -th local generator (on as "1" and off as "0"), $1 \leq n \leq N$
$u_n(t)$	The power output level when the n -th generator is on (Watt), $1 \leq n \leq N$
$s(t)$	The heat level obtained externally by natural gas (Watt)
$v(t)$	The power level obtained from electricity grid (Watt)

TABLE I: Notations of formulation.

unbalanced electricity demand in an on-demand manner. We let the spot price at time t from electricity grid be $p(t)$. We assume that $P_{\min} \leq p(t) \leq P_{\max}$. Again, we do not rely on any specific stochastic model on $p(t)$.

Local CHP Generators: The microgrid has N units of heterogeneous local CHP generators, each having an maximum power output capacity L_n . Without loss of generality, we assume $L_1 \geq L_2 \dots \geq L_N$. Other setting of local generators follows a common generator model [12], see Table I.

Co-generation and Heat Demand: The local CHP generators can simultaneously generate electricity and useful heat. Let the heat recovery efficiency for co-generation be η , *i.e.*, for each unit of electricity generated, η unit of useful heat can be supplied for free. Alternatively, without co-generation, heating can be generated separately using external natural gas, which costs c_g per unit time. Thus, ηc_g is the saving due to using co-generation to supply heat, provided that there is sufficient heat demand.

To ensure insightful results, we assume that $c_o + \frac{c_m}{L_N} < P_{\max} + \eta \cdot c_g$. This ensures that the minimum co-generation energy cost is cheaper than the maximum external energy price. If this is not the case, it is always optimal to obtain power and heat externally.

B. Problem Definition

The microgrid operational and generation cost in $[1, T]$ is given by

$$\text{Cost}(y, u, v, s) \triangleq \sum_{t=1}^T \left\{ p(t) \cdot v(t) + c_g \cdot s(t) + \sum_{n=1}^N [c_o \cdot u_n(t) + c_m \cdot y_n(t) + \beta [y_n(t) - y_n(t-1)]^+ \right\}, \quad (1)$$

which includes the cost of grid electricity, the cost of the external gas, and the operating and switching cost of local

CHP generators in the entire horizon $[1, T]$.

We formally define the **MCMP** as a mixed-integer programming problem, given electricity demand a , heat demand h , and grid electricity price p as time-varying inputs:

$$\min_{y, u, v, s} \text{Cost}(y, u, v, s) \quad (2)$$

$$\text{s.t. } u_n(t) \leq L_n \cdot y_n(t), \quad (3)$$

$$\sum_{n=1}^N u_n(t) + v(t) = a(t), \quad (4)$$

$$\eta \cdot \sum_{n=1}^N u_n(t) + s(t) = h(t), \quad (5)$$

$$u_n(t) - u_n(t-1) \leq R_{\text{up}}^n, \quad (6)$$

$$u_n(t-1) - u_n(t) \leq R_{\text{dw}}^n, \quad (7)$$

$$y_n(\tau) \geq \mathbf{1}_{\{y_n(t) > y_n(t-1)\}}, t+1 \leq \tau \leq t+T_{\text{on}}^n-1, \quad (8)$$

$$y_n(\tau) \leq 1 - \mathbf{1}_{\{y_n(t) < y_n(t-1)\}}, t+1 \leq \tau \leq t+T_{\text{off}}^n-1, \quad (9)$$

$$\text{var } y_n(t) \in \{0, 1\}, u_n(t), v(t), s(t) \in \mathbb{R}_0^+, \quad (10)$$

$$n \in [1, N], t \in [1, T],$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function and \mathbb{R}_0^+ represents the set of non-negative numbers. Specifically, constraint (3) captures the constraint of maximal output of the local generator. Constraints (4)-(5) ensure that the demands of electricity and heat energy balance, respectively. Constraints (6)-(7) capture the constraints of maximum ramping-up/down rates. Constraints (8)-(9) capture the minimum on/off period constraints.

III. MAIN RESULTS

In our previous work in [13], we present an algorithm CHASE for scheduling energy generation in microgrids with renewable energy sources and local generators with identical operating parameters. In this section, we generalize CHASE to the new heterogenous scenario we formulated in Sec. II, where microgrids employ different types of co-generation generators with heterogenous operating constraints including maximum output levels and ramping-up/-down rate limits.

In particular, at each time slot, we repeat the following three-step process:

STEP 1: Division of demand. By this process, the total demand is divided into layers of sub-demand $\sigma^{ly-n}(t) = (a^{ly-n}(t), h^{ly-n}(t))$ (see Fig. 2), such that n -th layer is assigned to be supplied by a specific generator with capacity L_n . The sub-demand $\sigma^{top}(t) = (a^{top}(t), h^{top}(t))$ is supplied externally.

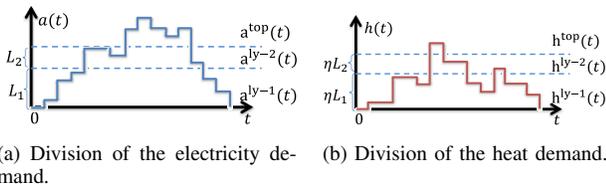


Fig. 2: The demand division sub-process. CHASE always deliver the demand to the generator with large capacity.

STEP 2: Deciding commitment variable $(y_n(t))$ for each sub-process. The decision process is illustrated in Fig 3.

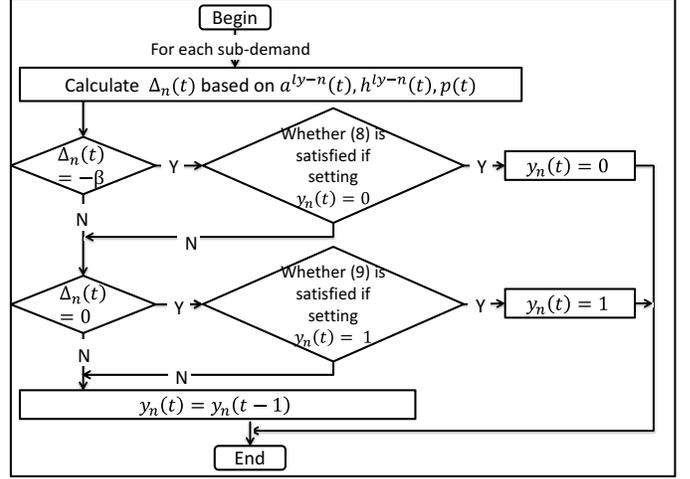


Fig. 3: The commitment variable $(y_n(t))$ decision sub-process

In Fig 3, let $\Delta_n \triangleq \psi_n(\sigma_n(t), 0) - \psi_n(\sigma_n(t), 1)$, and

$$\begin{aligned} \psi_n(\sigma_n(t), y_n(t)) &\triangleq \min_{u_n(t), v_n(t), s_n(t)} p(t)v_n(t) + c_g \cdot s_n(t) \\ &\quad + c_o(t) \cdot u_n(t) + c_m \cdot y_n(t) \\ \text{s.t. } &u_n(t) + v_n(t) = a_n(t). \\ &s_n(t) + \eta \cdot u_n(t) = h_n(t). \\ &u_n \leq y_n(t) \cdot L_n. \end{aligned}$$

STEP 3: Deciding dispatch variables $(u_n(t))$ decision sub-process. When $y_n(t)$ is determined, the optimal $u_n(t)$ can be decided by solving the following *single-time-slot* dispatch problem:

$$\begin{aligned} \min_{u_n(t), v_n(t), s_n(t)} & p(t)v_n(t) + c_g s_n(t) + c_o(t)u_n(t) \\ \text{s.t. } &u_n(t) + v_n(t) = a_n(t). \\ &s_n(t) + \eta \cdot u_n(t) = h_n(t). \\ &u_n \leq y_n(t) \cdot L_n. \\ &u_n(t) - u_n(t-1) \leq R_{\text{up}}^n. \\ &u_n(t-1) - u_n(t) \leq R_{\text{dw}}^n. \end{aligned}$$

In the above problem, $u_n(t-1)$ is the last time-slot dispatch variable, which was determined. The above problem is a linear programming problem, and can be solved with efficient algorithms [6]. Finally, we set $v(t) = a(t) - \sum u_n(t)$, $s(t) = h(t) - \eta \cdot \sum u_n(t)$.

By the the following theorem, we show that CHASE achieves a good performance guarantee against PD.

Theorem 1. *The cost of algorithm CHASE is at most $(3 - 2\alpha) \cdot \max(r_1 \cdot r_2)$ times of that achieved by PD. Here constants α, r_1, r_2 are given by:*

$$\alpha \triangleq (c_o + c_m/L_N)/(P_{\text{max}} + \eta \cdot c_g) \in (0, 1];$$

$$\begin{aligned} r_1 &\triangleq 1 + \max \left\{ \frac{P_{\text{max}} + c_g \cdot \eta - c_o}{L_N c_o + c_m} \max \{0, L_1 - R_{\text{up}}^{\min}\}, \right. \\ &\quad \left. \frac{c_o}{c_m} \max \{0, L_1 - R_{\text{dw}}^{\min}\} \right\}; \end{aligned}$$

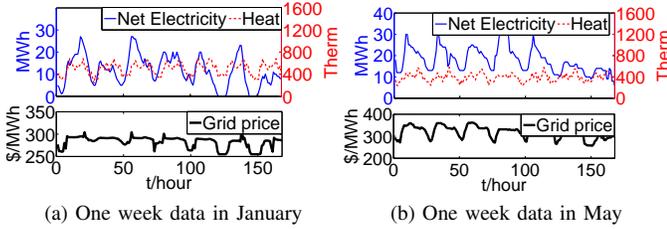


Fig. 4: Net electricity demand and heat demand for a typical week in January and May. The net demand is computed by subtracting the wind generation from the electricity demand.

$$r_2 \triangleq 1 + \frac{c_m \cdot T_{\text{on}}^{\text{max}} + L_1(P_{\text{max}} + c_g \cdot \eta)(T_{\text{on}}^{\text{max}} + T_{\text{off}}^{\text{max}})}{\beta}.$$

where $R_{\text{up}}^{\text{min}}$ and $R_{\text{dw}}^{\text{min}}$ are the minimal ramp up/down rate among all local generators, and $T_{\text{on}}^{\text{max}}$ and $T_{\text{off}}^{\text{max}}$ are the maximal minimum on/off time among all local generators.

The full proof is referred to our technical report [3].

For fast response generators², ramp limit constraints (6)(7) and minimum on/off constraints (8)(9) can be omitted (*i.e.*, $R_{\text{up}}^{\text{min}}, R_{\text{dw}}^{\text{min}} \rightarrow L_1$ and $T_{\text{on}}^{\text{max}}, T_{\text{off}}^{\text{max}} \rightarrow 0$). Under this scenario, r_1 and r_2 will decrease to 1 and the competitive ratio of CHASE becomes $3 - 2\alpha$, which is strictly smaller than 3, independent of input and system settings. Furthermore, we prove that under this special scenario, CHASE achieves the smallest *c.r.* among all the real-time scheduling algorithms. We note that we generalize CHASE to the version that can exploits a flexible time-window prediction in [3].

IV. EMPIRICAL EVALUATIONS

We evaluate the performance of our algorithm based on evaluations using operating traces of a pilot microgrid on the Bornholm island, Denmark [11], [1], [7]. Our objectives are three-fold: (i) evaluating the potential benefits of CHP and the ability of our algorithms to unleash such potential, (ii) corroborating the empirical performance of CHASE under various realistic settings, and (iii) understanding how the prediction error impacts the performance.

A. Parameters and Settings

Demand and Wind Trace: We use one-week trace in January and May in Bornholm island³, respectively. This trace is shown in Fig. 4.

Electricity and Natural Gas Prices: We use the corresponding grid electricity price (Fig. 4) and natural gas price data (Table. II) in Denmark.

	January, 2007	May, 2007
Natural Gas	0.0282 \$/kWh	0.0291 \$/kWh

TABLE II: The natural gas purchase price in Denmark.

Generator Model: We adopt generators with specifications the same as the one in [4]. We adopt ten generators with capacity $1MW \times 1$, $2MW \times 3$ and $5MW \times 6$. Other parameters

²Such as generators based on gas turbines and diesel engines.

³The data is generated by scaling down the western Denmark data by [11].

are shown as follows: $c_o = 0.051\$/KWh$, $c_m = 150\$/h$, $\eta = 1.8$, $\beta = 1400\$/h$, $T_{\text{on}} = T_{\text{off}} = 3h$ and $R_{\text{up}} = R_{\text{dw}} = 1MW/h$.

Local Heating System: We assume an on-demand heating system with capacity sufficiently large to satisfy all the heat demand by itself and without on-off cost or ramp limit. The efficiency of a heating system is set to 0.8 according to [2], and consequently we can compute the unit heat generation cost to be $c_g = 0.0179\$/KWh$.

Cost Benchmark: We use the cost incurred by using only external electricity, heating and wind energy (without CHP generators) as a benchmark. We evaluate the cost reduction due to our algorithm.

Comparisons of Algorithms: We compare three algorithms in our simulations. (1) our algorithm CHASE; (2) the Fixed Horizon Control (FHC) algorithm⁴; and (3) the Perfect Dispatch (PD) solution.

B. Potential Benefits of CHP

Purpose: The experiments in this subsection aim to answer two questions. First, what is the potential savings with microgrids? Second, what is the difference in cost-savings with and without the co-generation capability? In particular, we conduct two sets of experiments to evaluate the cost reductions of various algorithms. Both experiments have the same default settings, except that the first set of experiments (referred to as CHP) assumes the CHP technology in the generators are enabled, and the second set of experiments (referred to as NOCHP) assumes the CHP technology is not available, in which case the heat demand must be satisfied solely by the heating system.

Observations: First, looking at the performance of PD, we observe that PD achieves much more cost savings during May than during January. This is because the electricity price during May is very high, thus we can benefit much more from using the relatively-cheaper local generation as compared to using grid energy only. Moreover, PD achieves much more cost savings when CHP is enabled than when it is not during January. This is because, during January, the electricity price is relatively low and the heat demand is high. Hence, just using local generation to supply electricity is not economical. Rather, local generation becomes more economical only if it can be used to supply both electricity and heat together.

Second, CHASE performs consistently close to PD across inputs, even though the different settings have very different characteristics of demand and supply. In contrast, the performance of FHC depends heavily on the input characteristics. For example, FHC achieves some cost reduction during May and autumn when CHP is enabled, but achieves 0 cost reduction in all the other cases.

⁴In FHC, an estimate of the near future (*e.g.*, in a window of length w) is used to compute a tentative control trajectory that minimizes the cost over this time-window. All steps in the prediction window are implemented. In the next time slot, the prediction window shifts forward by w . Then, another control trajectory is computed based on the new future information, and again all steps are implemented. This process then continues. FHC represents the traditional scheduling approach based on perfect prediction.

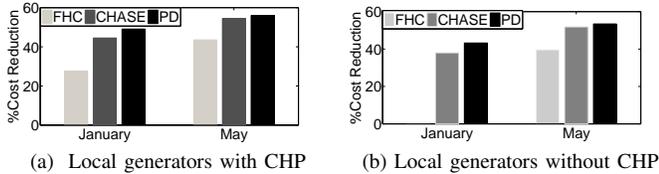


Fig. 5: Cost reductions for January and May .

C. Benefits of Perfect Prediction

Purpose: We compare the performances of CHASE to FHC and PD for different sizes of the perfect prediction window and show the results in Fig. 6. The vertical axis is the cost reduction as compared to the cost benchmark in Sec. IV-A and the horizontal axis is the size of prediction window, which varies from 0 to 20 hours.

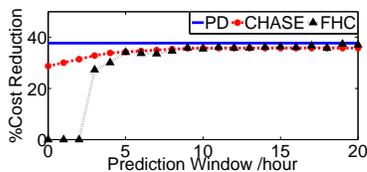


Fig. 6: Cost reduction as a function of perfect prediction window length.

Observations: We observe that the performance of our real time algorithm CHASE is already close to PD even when no or little perfect prediction information is available (*e.g.*, $w = 0, 1$, and 2). In contrast, FHC performs poorly when the prediction window is small. When w is large, both CHASE and FHC perform very well and their performance are close to PD when the prediction window w is larger than 15 hours.

An interesting observation is that it is more important to perform intelligent energy generation scheduling when there are no or little prediction information available. When there are abundant prediction information available, both CHASE and FHC achieve good performance and it is less critical to carry out sophisticated algorithm design.

D. Impacts of Prediction Error

Purpose: Previous experiments show that our algorithm have better performance if a larger time-window of accurate prediction input information is available. The input information in the prediction window include the wind station power output, the electricity and heat demand, and the central grid electricity price. In practice, these prediction information can be obtained by applying sophisticated prediction techniques based on the historical data. However, there are always prediction errors. For example, while the day-ahead electricity demand can be predicted within 2-3% range, the wind power prediction in the next hours usually comes with an error range of 20-50% [10]. Therefore, it is important to evaluate the performance of the algorithms in the presence of prediction error.

Observations: To achieve this goal, we evaluate CHASE with prediction window size of 3 hours. According to [10],

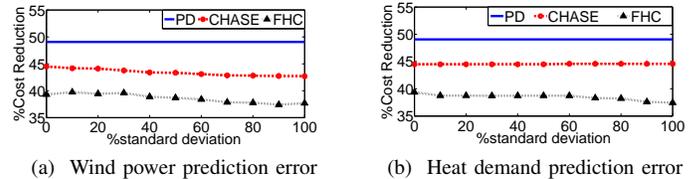


Fig. 7: Cost reduction as a function of the size prediction error.

the hour-level wind-power prediction-error in terms of the percentage of the total installed capacity usually follows beta distribution. Thus, in the prediction window, a zero-mean beta-distributed prediction error is added to the amount of wind power in each time-slot. We vary the standard deviation of the prediction error from 0 to 100% of the half of the total installed capacity. Similarly, a zero-mean beta distributed prediction error is added to the heat demand, and its standard deviation also varies from 0 to 100% of the half peak demand. We average 20 runs for each algorithm and show the results in Figs. 7a and 7b. As we can see, CHASE is fairly robust to the prediction error. Besides, the impact of the prediction error is relatively small when the prediction window size is small, which matches with our intuition.

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