Analog Network Coding in General SNR Regime: Performance of Network Simplification

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***Abstract*—A communication scenario where a source communicates with a destination over a directed layered relay network is considered. Each relay performs analog network coding where it scales and forwards the signals received at its input. In this scenario, we address the question: What portion of the maximum end-to-end achievable rate can be maintained if only a fraction of relay nodes available at each layer are used?**

We consider, in particular, the Gaussian diamond network and a class of symmetric layered networks. For these networks we provide upper bounds on additive and multiplicative gaps between the optimal analog network coding performance when all N relays in each layer are used and when only k such relays are are used, $k < N$ (network simplification). We show that asymptotically (in source power), the additive gap increases at most logarithmically with ratio N/k and the number of layers, and the corresponding multiplicative gap increases at most linearly with ratio N/k and is independent of the number of layers in the layered network. To the best of our knowledge, this work offers the first characterization of the performance of network simplification in general layered amplify-and-forward relay networks. Further, unlike most of the current approximation results that attempt to bound optimal rates either within an additive gap or a multiplicative gap, our results suggest a new rate approximation scheme that allows for the simultaneous computation of additive and multiplicative gaps.

I. Introduction

Analog network coding (ANC) extends to multihop wireless networks the idea of linear network coding [1], where an intermediate node sends out a linear combination of its incoming packets. In a wireless network, signals transmitted simultaneously by multiple sources add in the air. Thus, each node receives at its input a *noisy sum* of these signals, *i.e.* a linear combination of the received signals and noise. A communication scheme wherein each relay node merely amplifies and forwards this noisy sum is referred to as analog network coding [2], [3].

The rates achievable with ANC in layered relay networks is analyzed in [3], [4]. In [3], the achievable rate is computed under two assumptions: (A) each relay node scales the received signal to the maximum extent subject to its transmit power constraint, (B) the nodes in all L layers operate in the high-SNR regime. It is shown that the rate achieved under these assumptions approaches network capacity asymptotically (in source power). In [4] it is shown that even in the scenarios where the nodes in at most one layer do not satisfy these assumptions, achievable rates still approach the network capacity as the source power increases.

However, each relay node amplifying its received signal to the upper bound of its transmit power constraint results in suboptimal end-to-end performance of analog network coding in general, as we show in [5], [6] and was also previously indicated in [7]. Further, even in low-SNR regimes amplify-and-forward (AF) relaying can be capacity-achieving relay strategy in some scenarios, [8]. Therefore, we are concerned with characterizing the performance of analog network coding in general layered networks, without the above two assumptions on input signal scaling factors and received SNRs.

Analyzing the performance of analog network coding without such assumptions however, results in a computationally intractable problem in general [4], [5]. Therefore, to compute the maximum achievable ANC rate in general communication scenarios, in [5] we establish a result that significantly reduces the computational complexity of this problem. Further in [6], we propose a greedy scheme to bound from below the optimal rate achievable with analog network coding in general layered networks. For a large class of symmetric layered networks, these two results allow us to exactly compute the optimal ANC rates that cannot be computed using existing approaches based on assumptions (A) and (B). Further for general layered relay networks, these results lead to a tighter approximation of the optimal ANC rates in a computationally efficient manner.

This paper introduces another approach to reduce the computational complexity of approximating the maximum achievable ANC rate in general layered networks. The proposed approach is based on the notion of *network simplification*, introduced in [9] to characterize fraction of the capacity of the Gaussian N-relay diamond network when only k out of N available relay nodes are used. Previously, in *cooperative communication* literature [10]–[12], network simplification is used in AF relay networks to characterize achievable rates. However, such prior work used network simplification in a restricted sense (selecting the *best* single relay node among N relays) and considered simple network topologies (the source communicating with the destination via a single relay node). In contrast, we provide the optimal ANC rate characterization in much general communication scenarios where any k out of N relay nodes in a layer are used ($k < N$) and the source communicates with the destination over L of layers of relay nodes ($L \geq 1$). In this sense, to the best of our knowledge, this is the first work to characterize the performance of network simplification in such general layered AF relay networks.

We show that in the Gaussian N-relay diamond network and a class of symmetric layered networks, network simplification allows us to maintain achievable rates within small additive and multiplicative gaps of the maximum ANC rate achievable

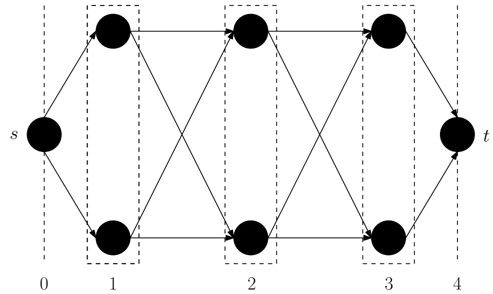

Fig. 1. Layered network with 3 relay layers between source s and destination t. Each layer contains two relay nodes.

when all relays in every layer are used. Along with our previous results in [5], [6], where we establish that analog network coding is a capacity achieving strategy in some communication scenarios, the results in this paper establish that in those scenarios using fewer nodes in each layer, we can still maintain achievable rates within small additive and multiplicative gaps to the capacity. Further our results indicate that in general layered networks, the maximum achievable ANC rate can be tightly approximated with much lower computational complexity than with existing schemes.

In this paper we provide the summary of our work. We have omitted most proofs or give only brief outlines. The details can be found in our arXiv submission [13].

Organization: In Section II we introduce a general wireless layered relay network model and formulate the problem of maximum rate achievable with ANC in such a network. Section III addresses the performance of network simplification in the Gaussian N-relay diamond network and computes additive and multiplicative gaps between the maximum ANC rates achievable when N and k ($k < N$), relays are used. In Section IV we consider a class of symmetric layered networks and compute additive and multiplicative gaps between the optimal ANC rates obtained with and without network simplification. Section V concludes the paper.

II. System Model

Consider a $(L+2)$-layer wireless network with directed links[1]. Source s is at layer '0', destination t is at layer '$L+1$', and the relay nodes from set R are arranged in L layers between them. The l^{th} layer contains n_l relay nodes, $\sum_{l=1}^{L} n_l = |R|$. An instance of such a network is given in Figure 1. Each node is assumed to have a single antenna and operate in full-duplex mode.

At instant n, the channel output at node $i, i \in R \cup \{t\}$, is

$$y_i[n] = \sum_{j \in \mathcal{N}(i)} h_{ji} x_j[n] + z_i[n], \quad -\infty < n < \infty, \tag{1}$$

where $x_j[n]$ is the channel input of node j in neighbor set $\mathcal{N}(i)$ of node i. In (1), h_{ji} is a real number representing the channel gain along the link from node j to node i. It is assumed to be fixed (for example, as in a single realization of a fading process) and known throughout the network. Source symbols $x_s[n], -\infty < n < \infty$, are i.i.d. Gaussian random variables with zero mean and variance P_s that satisfy an average source power constraint, $x_s[n] \sim \mathcal{N}(0, P_s)$. Further, $\{z_i[n]\}$ is a sequence (in n) of i.i.d. Gaussian random variables with $z_i[n] \sim \mathcal{N}(0, \sigma^2)$. We assume that z_i are independent of the input signal and of each other. We also assume that the i^{th} relay's transmit power is constrained as:

$$E[x_i^2[n]] \le P_i, \quad -\infty < n < \infty \tag{2}$$

In analog network coding each relay node amplifies and forwards the noisy signal sum received at its input. More precisely, relay node $i, i \in R$, at instant $n+1$ transmits the scaled version of $y_i[n]$, its input at time instant n, as follows

$$x_i[n+1] = \beta_i y_i[n], \quad 0 \le \beta_i^2 \le \beta_{i,max}^2 = P_i / P_{R,i}, \tag{3}$$

where $P_{R,i}$ is the received power at node i and choice of scaling factor β_i satisfies the power constraint (2).

In the class of layered networks shown in Figure 1 where the nodes in a layer communicate only with the nodes in the next immediate layer, all copies of a source signal and a noise symbol introduced at a node, traveling along different paths, arrive at the destination with the same respective time delays. Therefore, the output of the source-destination channel is free of intersymbol interference. This simplifies the relation between input and output of the channel and allows us to omit the time-index while denoting the input and output signals.

Using (1) and (3), the input-output channel between source s and destination t in a layered network can be written as

$$y_t = h_s x_s + \sum_{l=1}^{L} \sum_{j=1}^{n_l} h_{lj} z_{lj} + z_t, \tag{4}$$

where *modified* channel gains h_s and h_{lj} are defined in [5], where we also illustrate the derivation of the source-destination channel expression in (4) for a specific layered network.

Problem Formulation: For a given network-wide scaling vector $\boldsymbol{\beta} = (\beta_{li})_{1 \le l \le L, 1 \le i \le n_l}$, the achievable rate for the channel in (4) with i.i.d. Gaussian input is ([3]–[5]):

$$I(P_s, \boldsymbol{\beta}) = (1/2) \log\big(1 + SNR_t\big), \tag{5}$$

where SNR_t, the signal-to-noise ratio at destination t is:

$$SNR_t = \frac{P_s}{\sigma^2} \frac{h_s^2}{1 + \sum_{l=1}^{L} \sum_{j=1}^{n_l} h_{lj}^2} \tag{6}$$

The maximum ANC rate $I_{ANC}(P_s)$ achievable in a given layered network with i.i.d. Gaussian input is defined as the maximum of $I(P_s, \boldsymbol{\beta})$ over all feasible $\boldsymbol{\beta}$ subject to per relay transmit power constraint (3). In other words:

$$I_{ANC}(P_s) \overset{def}{=} \max_{\boldsymbol{\beta}: 0 \le \beta_{li}^2 \le \beta_{li,max}^2} I(P_s, \boldsymbol{\beta}) \tag{7}$$

Given the monotonicity of the $\log(\cdot)$ function, we have

$$\boldsymbol{\beta}_{opt} = \underset{\boldsymbol{\beta}: 0 \le \beta_{li}^2 \le \beta_{li,max}^2}{\operatorname{argmax}} I(P_s, \boldsymbol{\beta}) = \underset{\boldsymbol{\beta}: 0 \le \beta_{li}^2 \le \beta_{li,max}^2}{\operatorname{argmax}} SNR_t \tag{8}$$

[1]The layered networks with bidirectional links can be addressed with the *signal subtraction* notion we introduced in [14]. However, for the ease of presentation we do not discuss such networks in this paper.

Therefore in the rest of this paper, we concern ourselves mostly with maximizing the received SNRs.

In [5] we discussed the computational complexity of exactly solving the problem in (7) or the one in (8). Further, we also introduced a key result [5, Lemma 2] that reduces the computational complexity of the problem of computing $\boldsymbol{\beta}_{opt}$ by computing it layer-by-layer as a solution of a cascade of subproblems. However, each of these subproblems itself is computationally hard for general network scenarios as it involves maximizing the ratio of *posynomials* [15], [16], which is known to be computationally intractable in general [16]. Therefore in this paper, based on the notion of network simplification [9], we introduce an approach to reduce the computational complexity of solving each of these subproblems exponentially by selecting a subset of k_l relays among the set of n_l relays in the l^{th} layer. The proposed scheme in conjunction with [5, Lemma 2] leads to further reduction in the complexity of approximating the solution of (7) while still maintaining achievable rates within small additive and multiplicative gaps from the optimal performance obtained without network simplification.

In the following, we first motivate this approach by approximating the maximum achievable ANC rate for the Gaussian N-relay diamond networks, and then discuss its performance for a class of symmetric layered networks.

III. Analog Network Coding in the Diamond Network: Performance of Network Simplification

The diamond network, as in Figure 2, can be considered as a layered network with only one layer of relay nodes. Then using the definitions of modified channels gains and (6), the SNR at destination t for any scaling vector $\boldsymbol{\beta}$ is

$$SNR_t = \frac{P_s}{\sigma^2}\frac{(\sum_{i=1}^{N} h_{si}\beta_i h_{it})^2}{1+\sum_{i=1}^{N}\beta_i^2 h_{it}^2} \tag{9}$$

Therefore, using (7) the problem of computing the maximum ANC rate for this network can be formulated as

$$\max_{\boldsymbol{\beta}^2 \le \boldsymbol{\beta}^2_{max}} SNR_t, \tag{10}$$

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_N)$ and $\boldsymbol{\beta}_{max} = (\beta_{1,max} \cdots, \beta_{N,max})$ with $\beta^2_{i,max} = P_i/(h^2_{si}P_s + \sigma^2), i \in \mathcal{N}, \mathcal{N} = \{1, \ldots, N\}$.

In [17] it is observed that at high rate (or high SNR in the present setting) we should be able to approximate the maximum achievable rate within an additive gap and at low rate (or low SNR) we should be able to do so within a multiplicative gap. In this paper, we are particularly concerned with the variation of the destination SNR with the source power. Therefore for the ease of presentation, we compute the additive gap for large P_s (high-SNR regime) and the multiplicative gap for small P_s (low-SNR regime).

Let $R_N = (1/2)\log(1 + SNR^{opt}_{t,N})$ denote the optimal end-to-end ANC rate achieved by using all N relays in the diamond network. Similarly, let $R_k = (1/2)\log(1+SNR^{opt}_{t,k})$ denote the optimal end-to-end ANC rate achieved by using the rate-optimal k-subset of N available relays in the diamond network. Our first main result in Lemma 1 provides the upper-bounds on additive gap $R_N - R_k$ and multiplicative gap R_N/R_k, for such networks.

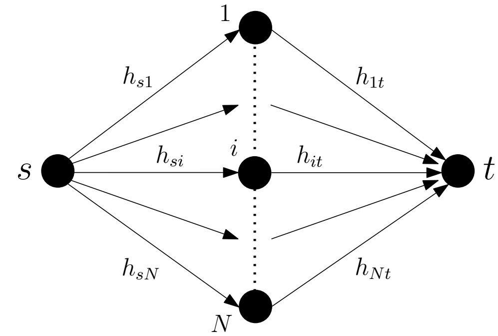

Fig. 2. A diamond network with N relay nodes.

Lemma 1: For the Gaussian N-relay diamond network, the additive gap (in high-SNR regime) and the multiplicative gap (in low-SNR regime) between the optimal performance of analog network coding obtained with and without network simplification is bounded from above, respectively, as

$$\text{(for } P_s \to \infty)\ R_N - R_k \le \frac{1}{2\ln 2}\min\{2\ln(N/k) + \ln\alpha_1,\ 2(H_{N-1} - H_{k-1}) + (N-k)\ln\alpha_1\}$$

$$\text{(for } P_s \to 0) \qquad \frac{R_N}{R_k} \le \frac{N}{k}\alpha_2\left(1 + \frac{\gamma}{k}\right),$$

where parameters α_1 and α_2 characterize the asymmetry in the network and in general $\alpha_1, \alpha_2 \ge 1$, and

$$\alpha_1 = \frac{(h^2_{si,max}h_{it,max})^2}{(h^2_{si,min}h_{it,min})^2},$$

$$\alpha_2 = \frac{(h_{si,max}h_{it,max})^2}{(h_{si,min}h_{it,min})^2}\ \frac{h^2_{si,max}+\sigma^2}{h^2_{si,min}+\sigma^2},$$

$$\gamma = \frac{1}{(\beta^{max}_{i,max}h_{it,max})^2},$$

and H_n is Harmonic number, $\lim_{n\to\infty} H_n \sim \ln n + \gamma$ and γ is *Euler-Mascheroni* constant [18].

Example 1: Consider the Gaussian N-relay diamond network in the symmetric configuration, in which the channel gains to the relays are equal ($h_{si} = h, i \in \mathcal{N}$), the channel gains from the relays are equal ($h_{it} = g, i \in \mathcal{N}$), and in general $h \neq g$. Also assume that the transmit power constraint on each relay is the same, *i.e.* $E[x_i^2] \le P, i \in \mathcal{N}$. In this setting $\beta^2_{i,max} = \beta^2 = P/(h^2P_s + \sigma^2)$ and $\gamma = 1/(\beta g)^2$. Also, $\alpha_1 = \alpha_2 = 1$. Therefore, using Lemma 1 we obtain the following upper bounds on additive and multiplicative gaps between R_N and R_k when $\beta g \ge 1$:

$$R_N - R_k \le \frac{1}{\ln 2}(H_{N-1} - H_{k-1}) \overset{(a)}{\sim} \log\frac{N}{k}$$

$$\frac{R_N}{R_k} \le \frac{N}{k}(1+\gamma/k) \le \frac{N}{k}(1+\gamma),$$

where (a) follows from the asymptotic behavior of H_n. ■

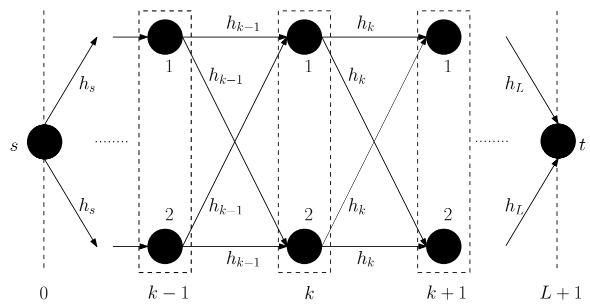

Fig. 3. An ECGAL network of $L+2$ layers, with source s in layer '0', destination t in layer '$L+1$', and L relay layers with two nodes each. The channel gains along all links between adjacent layers are equal.

IV. Performance of Network Simplification in General Layered Networks

In this section we analyze the performance of network simplification in a class of symmetric layered networks with more than one layer of relays between source and destination and each relay performing analog network coding on its input signal. The particular class of symmetric networks we consider here are defined such that channel gains along all links between the nodes in two adjacent layers are equal. This is a generalization of the symmetric diamond network configuration considered in Example 1 above. We introduced such networks in [5] and called them *"Equal Channel Gains between Adjacent Layers (ECGAL)"* networks. Figure 3 provides an illustration of such networks.

For the ease of presentation, consider ECGAL networks where each layer of relay nodes has N relays and all relay nodes have the same transmit power constraint $EX^2 \le P$.

Using Lemma 2 in [5], we can obtain the optimal solutions $\boldsymbol{\beta}^N_{opt}$ and $\boldsymbol{\beta}^k_{opt}$ of problem (8) for an ECGAL network when all N relays and when only $k, k<N$, relays in each layer are used, respectively. For these optimal network-wide scaling vectors, we can then compute the corresponding optimal SNRs at the destination, denoted as $SNR^{opt}_{t,N}$ and $SNR^{opt}_{t,k}$, respectively. The ratio of these optimal SNRs is given as

$$\frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} = \left(\frac{N}{k}\right) \frac{\sum\limits_{l=1}^{L} \frac{1}{\left(\prod\limits_{j=1}^{l-1}(k\beta^k_j h_j)\right)^2} + \frac{k}{\left(\prod\limits_{l=1}^{L}(k\beta^k_l h_l)\right)^2}}{\sum\limits_{l=1}^{L} \frac{1}{\left(\prod\limits_{j=1}^{l-1}(N\beta^N_j h_j)\right)^2} + \frac{N}{\left(\prod\limits_{l=1}^{L}(N\beta^N_l h_l)\right)^2}} \quad (11)$$

Let $R_N = (1/2)\log(1+SNR^{opt}_{t,N})$ denote the optimal end-to-end ANC rate achieved by using all N relays in every layer of an ECGAL network. Similarly, let $R_k = (1/2)\log(1 + SNR^{opt}_{t,k})$ denote the optimal end-to-end ANC rate achieved by using any k out of N available relays in every layer of an ECGAL network. Arguing as in Section III, in the following we compute the additive gap $R_N - R_k$ for large P_s and the multiplicative gap R_N/R_k for small P_s.

Before we discuss the upper-bounds on the additive and multiplicative gaps for ECGAL networks with arbitrary number of relay layers, consider the following example where we compute such bounds for an ECGAL network with two layers of relay nodes ($L=2$) for any N and k.

Example 2: Consider an ECGAL network with two layers ($L=2$) of relay nodes between the source and the destination. Using (11), we have for this network

$$\frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} = \left(\frac{N}{k}\right) \frac{1 + \frac{1}{k^2 h_1^2 \beta_1^2} + \frac{1}{k^3 h_1^2 h_2^2 \beta_1^2 (\beta_2^k)^2}}{1 + \frac{1}{N^2 h_1^2 \beta_1^2} + \frac{1}{N^3 h_1^2 h_2^2 \beta_1^2 (\beta_2^N)^2}}$$

Substituting for β_1, β_2^N, and β_2^k results in

$$(\text{for } P_s \to \infty)\ \frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} = \left(\frac{N}{k}\right)^2 \frac{1 + \frac{h_2^2}{k h_1^2} + \frac{\sigma^2}{k^2 h_1^2 P}}{1 + \frac{h_2^2}{N h_1^2} + \frac{\sigma^2}{N^2 h_1^2 P}}$$

$$(\text{for } P_s \to 0)\ \frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} = \left(\frac{N}{k}\right) \frac{1 + \frac{\sigma^2}{k^2 P}\left(\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{\sigma^2}{k h_1^2 h_2^2 P}\right)}{1 + \frac{\sigma^2}{N^2 P}\left(\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{\sigma^2}{N h_1^2 h_2^2 P}\right)}$$

These expressions for $\frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}}$ result in the following upper bounds on the corresponding additive and multiplicative gaps:

$$(P_s \to \infty) R_N - R_k \le \left[\log\frac{N}{k} + \frac{1}{2}\log\frac{1 + \frac{h_2^2}{k h_1^2} + \frac{\sigma^2}{k^2 h_1^2 P}}{1 + \frac{h_2^2}{N h_1^2} + \frac{\sigma^2}{N^2 h_1^2 P}}\right]$$

$$(P_s \to 0) \qquad \frac{R_N}{R_k} \le \left(\frac{N}{k}\right) \frac{1 + \frac{\sigma^2}{k^2 P}\left(\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{\sigma^2}{k h_1^2 h_2^2 P}\right)}{1 + \frac{\sigma^2}{N^2 P}\left(\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{\sigma^2}{N h_1^2 h_2^2 P}\right)}$$

for two layer ECGAL networks with any N and k. ■

However, for general ECGAL networks it is analytically hard to compute such upper-bounds on the additive and multiplicative gaps between the optimal end-to-end performances with and without network simplification for arbitrary N and k. Therefore, in the following we analyze the scaling behavior of such upper bounds with large N and k.

Consider the asymptotic behavior of $\frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}}$ with P_s. We have for large N and k:

$$(P_s \to \infty)\ \prod_{i=1}^{l} (\beta_i^N)^2 = \frac{P/(h_s^2 P_s)}{N^{2(l-1)} \prod_{i=1}^{l-1} h_i^2}, 1 \le l \le L \quad (12)$$

$$\prod_{i=1}^{l} (\beta_i^k)^2 = \frac{P/(h_s^2 P_s)}{k^{2(l-1)} \prod_{i=1}^{l-1} h_i^2}, 1 \le l \le L$$

$$(P_s \to 0)\ \prod_{i=1}^{l} (\beta_i^N)^2 = \frac{P/\sigma^2}{N^{2(l-1)-1} \prod_{i=1}^{l-1} h_i^2}, 1 \le l \le L \quad (13)$$

$$\prod_{i=1}^{l} (\beta_i^k)^2 = \frac{P/\sigma^2}{k^{2(l-1)-1} \prod_{i=1}^{l-1} h_i^2}, 1 \le l \le L$$

Then, from (11)-(13) we obtain for large N and k:

$$(P_s \to \infty) \quad \frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} \sim \left(\frac{N}{k}\right)^2 \left[1 + a\left(\frac{1}{k} - \frac{1}{N}\right)\right], \quad (14)$$

$$(P_s \to 0) \quad \frac{SNR^{opt}_{t,N}}{SNR^{opt}_{t,k}} \sim \left(\frac{N}{k}\right) \left[1 + b\left(\frac{1}{k^2} - \frac{1}{N^2}\right)\right], \quad (15)$$

where $a = h_L^2 \sum_{i=1}^{L-1} 1/h_i^2$ and $b = \frac{\sigma^2}{P}(1/h_1^2 + 1/h_L^2)$.

Let $\overline{R}_N$ denote the asymptotic (in N) value of R_N. Similarly, let $\overline{R}_k$ denote the asymptotic (in k) value of R_k. Then, using (14) and (15), the asymptotic behavior of the additive and multiplicative gaps is characterized in the following lemma that is the second main result of this paper.

Lemma 2: For ECGAL networks, the asymptotic (in N and k) additive and multiplicative gaps between the optimal ANC performance obtained with and without network simplification are bounded from above, respectively, as

$$\begin{aligned}\overline{R}_N - \overline{R}_k &\leq \left\{ \log \frac{N}{k} + \frac{1}{2} \log \left[1 + a \left(\frac{1}{k} - \frac{1}{N} \right) \right] \right\} \\ &\leq \left[\log \frac{N}{k} + \frac{1}{2} \log(1+a) \right], \\ \frac{\overline{R}_N}{\overline{R}_k} &\leq \left(\frac{N}{k} \right) \left[1 + b \left(\frac{1}{k^2} - \frac{1}{N^2} \right) \right] \\ &\leq \left(\frac{N}{k} \right) (1+b),\end{aligned}$$

where $a = h_L^2 \sum_{i=1}^{L-1} 1/h_i^2$ and $b = \frac{\sigma^2}{P}(1/h_1^2 + 1/h_L^2)$ are constants depending on the system parameters.

Remark 1: The bounds in Lemma 2 imply that asymptotically the additive gap increases at most logarithmically with the number of layers in the network and the multiplicative gap is independent of the number of layers. These results are in agreement with the corresponding results obtained for the diamond network (with a single relay layer) in Section III. For instance, for the network in *Example 1*, using the results of this section we obtain

$$\begin{aligned}\overline{R}_N - \overline{R}_k &\leq \log \frac{N}{k} \\ \frac{\overline{R}_N}{\overline{R}_k} &\leq \frac{N}{k} \left(1 + \frac{\sigma^2}{P g^2} \right),\end{aligned}$$

with $L = 1, a = 0$ and $h_1 = g$. These bounds coincide exactly with the corresponding asymptotic (in N and k) upper-bounds in *Example 1*. This indicates that the bounds in Lemma 2 are actually asymptotically tight for the diamond network.

Remark 2: The bounds in Lemma 2 are obtained for general network configurations, without any restriction on channel gains and number of layers. Therefore for any specific network configuration, these bounds are in general loose compared to the bounds obtained from the first principles for such networks. For instance, for the networks in Examples 1 and 2, Lemma 2 provides bounds that are looser than the corresponding bounds obtained for any N and k in these examples.

V. Conclusion and Future Work

Computing the maximum end-to-end ANC rate in general layered relay networks is an important but computational intractable problem. We introduce an approach based on the notion of *network simplification* to approximate the optimal ANC rate within small additive and multiplicative gaps in the Gaussian N-relay diamond network and a class of symmetric layered networks while simultaneously reducing the computational complexity of solving this problem. To the best of our knowledge, this work provides the first characterization of the performance of network simplification in general layered AF networks. Also, our results suggest a new approach to approximate the optimal ANC rates while allowing for the computation of the additive and multiplicative gaps simultaneously. In future, we plan to extend this work to general layered networks.

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References

[1] S. -Y. R. Li, R. W. Yeung, and N. Cai, "Linear network coding," *IEEE Trans. Inform. Theory,* vol. IT-49, February 2003.

[2] S. Katti, S. Gollakotta, and D. Katabi, "Embracing wireless interference: analog network coding," *Proc. SIGCOMM,* Kyoto, Japan, August, 2007.

[3] I. Marić, A. Goldsmith, and M. Médard, "Analog network coding in the high-SNR regime," *Proc. IEEE WiNC 2010,* Boston, MA, June 2010.

[4] B. Liu and N. Cai, "Analog network coding in the generalized high-SNR regime," *Proc. IEEE ISIT 2011,* St. Petersburg, Russia, July 2011.

[5] S. Agnihotri, S. Jaggi, and M. Chen, "Analog Network Coding in General SNR Regime," *Proc. IEEE ISIT 2012,* Cambridge, MA, July 2012. Longer version available at *arXiv:1202.0372*.

[6] S. Agnihotri, S. Jaggi, and M. Chen, "Analog network coding in general SNR regime: performance of a greedy scheme," *Proc. IEEE NetCod 2012,* Cambridge, MA, June 2012.

[7] B. Schein, *Distributed Coordination in Network Information Theory*. PhD thesis, Massachusetts Institute of Technology, 2001.

[8] K. S. Gomadam and S. A. Jafar, "Optimal relay functionality for SNR maximization in memoryless relay networks," *IEEE JSAC,* vol. 25, February 2007.

[9] C. Nazaroglu, A. Özgür, and C. Fragouli, "Wireless network simplification: the Gaussian N-relay Diamond Network," *Proc. IEEE ISIT 2011,* St. Petersburg, Russia, July 2011.

[10] A. Bletas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE JSAC,* vol. 24, March 2006.

[11] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Comm.,* vol. 6, August 2007.

[12] J. Cai, X. Shen, J. W. Mark, and A. S. Alfa, "Semi-distributed user relaying algorithm for amplify-and-forward wireless relay networks," *IEEE Trans. Wireless Comm.,* vol. 7, April 2008.

[13] S. Agnihotri, S. Jaggi, and M. Chen, "Analog network coding in general SNR regime: performance of network simplification," *arXiv:1204.2150*.

[14] S. Agnihotri, S. Jaggi, and M. Chen, "Amplify-and-Forward in wireless relay networks," *Proc. IEEE ITW 2011,* Paraty, Brazil, October 2011.

[15] S. Boyd, S. -J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optim. Eng.,* vol. 8, April 2007.

[16] M. Chiang, *Geometric Programming for Communication Systems*. now Publishers Inc., Boston, 2005.

[17] U. Niesen and S. Diggavi, "The approximate capacity of the Gaussian N-relay diamond network," *Proc. IEEE ISIT 2011,* St. Petersburg, Russia, July 2011.

[18] D. Knuth, *The Art of Computer Programming, Volume 1: Fundamental Algorithms*. Addison-Wesley, 3rd ed., 1997.