

Optimizing Two-Truck Platooning with Deadlines

Wenjie Xu, Titing Cui, and Minghua Chen, *Fellow, IEEE*

Abstract—We study a transportation problem where two heavy-duty trucks travel across the national highway from separate origins to destinations, subject to individual deadline constraints. Our objective is to minimize their total fuel consumption by jointly optimizing path planning, speed planning, and platooning configuration. Such a two-truck platooning problem is pervasive in practice yet challenging to solve due to hard deadline constraints and enormous platooning configurations to consider. We first leverage a unique problem structure to significantly simplify platooning optimization and present a novel formulation. We prove that the two-truck platooning problem is *weakly* NP-hard and admits a Fully Polynomial Time Approximation Scheme (FPTAS). The FPTAS can achieve a fuel consumption within a ratio of $(1 + \epsilon)$ to the optimal (for any $\epsilon > 0$) with a time complexity polynomial in the size of the transportation network and $1/\epsilon$. These results are in sharp contrast to the general multi-truck platooning problem, which is known to be APX-hard and repels any FPTAS. As the FPTAS still incurs excessive running time for large-scale cases, we design an efficient dual-subgradient algorithm for solving large-/national-scale instances. It is an iterative algorithm that always converges. We prove that each iteration only incurs polynomial-time complexity, albeit it requires solving an integer linear programming problem optimally. We characterize a condition under which the algorithm generates an optimal solution and derive a posterior performance bound when the condition is not met. Extensive simulations based on real-world traces show that our joint solution of path planning, speed planning, and platooning saves up to 24% fuel as compared to baseline alternatives.

Index Terms—Energy efficiency, Timely truck transportation, Path planning, Speed planning, Platooning, Algorithm, Optimization

I. INTRODUCTION

Heavy-duty trucks, transporting freights across national highways, play a critical role in society’s economic development. It is reported that the U.S. trucking industry hauled around 80% of all freight tonnage and brought in 792 billion U.S. dollars as revenue in 2019 [1]. This number would rank 18th if measured against the country’s GDP [2]. Meanwhile, the trucking industry is also responsible for a large fraction of the world’s energy consumption and greenhouse gas emissions. For example, in the U.S., with only 4% of the vehicle population, heavy-duty trucks account for 18% energy

consumption of the entire transportation sector and 5% of the greenhouse gas emission [1]. In addition, fuel cost corresponds to 24% of the total operational cost in the trucking industry in 2019 [3]. These alerting observations, together with that the global freight activity is predicted to increase by a factor of 2.4 by 2050 [4], make it critical to develop effective solutions for improving fuel efficiency of truck transportation.

Truck platooning, where two or more trucks travel in convoy with small headway, is a promising fuel-saving approach. It is made possible in practice by the novel semi-automated driving technology, also referred to as the Cooperative Adaptive Cruise Control (CACC) system [5]–[7]. Similar to what racing cyclists exploit, trucks traveling in a platoon experience a reduction in air drag, which translates into lower fuel consumption. Many studies, e.g., [8]–[12], suggest up to 20% fuel saving for the non-leading vehicles in a platoon. Meanwhile, platooning also allows vehicles to drive closer than otherwise at the same speed, improving traffic throughput [13] and even safety. To date, platooning has received significant attention from private fleets and commercial carriers [14].

However, it is algorithmically non-trivial to capitalize the platooning potential to minimize the convoy’s total fuel consumption. First, it requires exploring three distinct design spaces: path planning, speed planning, and platooning configuration. Path planning and speed planning are well-recognized mechanisms for saving fuels by optimizing the driving paths and the speed profiles over individual road segments. Real-world studies show up to 21% fuel saving by driving at fuel-economic speeds along an optimized path [15]. Meanwhile, platooning configurations define the road segments for platooning and the trucks to participate in each platoon. In other words, it describes where and when a set of the trucks form and end a platoon. It adds another layer of combinatorial structure to the design space.

Second, the problem becomes even more complicated when we consider transportation deadlines for individual trucks. Freight delivery with time guarantee is common in truck operation; see examples and discussions in [16], [17]. As estimated by the U.S. Federal Highway Administration (FHWA) [18], an unexpected delay can increase freight cost by 50% to 250%. The operator needs to keep track of the time spent so far and the remaining time budget for individual trucks, when optimizing over the design space discussed above, to ensure on-time arrival at destinations. Such a 4-way coupling among path planning, speed planning, platooning configuration, and deadline creates a unique challenge. Indeed, the problem of minimizing fuel consumption for multiple platooning trucks is shown to be APX-hard in general [19], and it admits no polynomial-time algorithm with a fuel consumption within a constant ratio to the minimum, unless $\mathbf{P} = \mathbf{NP}$ [20], [21].

Existing studies. Previous researches on truck platooning

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W. Xu is with École Polytechnique Fédérale de Lausanne, Lausanne CH-1015, Switzerland.

T. Cui is with Joseph M. Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260, USA.

M. Chen is with School of Data Science, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Kowloon, Hong Kong.

The first two authors are co-primary authors. A part of the work was done when all the three authors were with The Chinese University of Hong Kong. Corresponding author: Minghua Chen (minghua.chen@cityu.edu.hk).

TABLE I: Summary of existing studies on minimizing fuel consumption for truck transportation with platooning.

Existing Study	Path Planning	Speed Planning	Platooning	Deadline	Approach	Bound
[21]–[24]	7	X	7	X	Convex optimization	7
[25]	7	X	X	X	Distributed method	7
[26]	X	7	X	7	Hub heuristic	7
[20]	X	7	X	X	Integer programming (IP)	7
[27]	X	X	X	X	K-set clustering + IP	7
[28]	7	7	X	X	Mixed integer programming	X
[29]	X	7	X	X	Iterative heuristic	7
This work	X	X	X	X	Dual-based optimization	Posterior

: Entries in the column represent whether there is a guaranteed performance gap to the optimal. Usually, performance bounds are independent to the solutions obtained by the algorithm and can be computed beforehand. Meanwhile, posterior performance bounds are solution dependent and can be computed after the solution is obtained.

optimization mostly consider either speed planning for given Table I summarizes existing studies on fuel consumption paths or path planning with a set of constant speeds. Assumed minimization in truck platooning. Contributions. In this paper, we focus on a prevalent all the trucks drive on their shortest or pre-specified paths, the authors of [21], [22], [24] obtain the optimal speed profiles for two-truck platooning scenario where two heavy-duty trucks for each truck by solving a series of convex optimization travel across the national highway from separate origins to problems. The authors in [23] and [30] use a similar technique destinations [40]–[44]. Two-truck platooning is not only the to solve a fixed-path platooning problem. When the traveling mainstream of current platooning practice [42], but it also speed on each road segment is fixed, the path planning incurs minimum safety concern for the surrounding traffic the platooning problem is often formulated as an integer sharing the roads [40] [43] [41]. To our best knowledge, this is the first work in the literature linear programming (ILP) problem [20], [27], [31]–[33]. In order to minimize total fuel consumption for truck transportation [20], [27], [31], heuristics such as the best-pair algorithms by simultaneously optimizing path planning, speed planning, are proposed to solve the ILP in the platooning problem, and platooning configuration, subject to individual deadline Meanwhile, [32] and [33] apply genetic algorithms to path constraints. We also consider departure coordination, where planning. [28] employs a mixed-integer programming model to trucks can strategically wait at the origins for proper schedule study the platooning of trucks along an identical path but with alignment to form an efficient platoon later. We make the different time windows and maximum platooning length. The following contributions. The studies in [27], [34] consider the platooning optimization with In Sec. II, we present a novel formulation for the two-truck platooning problem of minimizing fuel consumption discrete speeds for individual edges, which can be modeled as a path planning problem by introducing multiple parallel edges subject to individual deadline constraints, taking into account between the starting and ending nodes of original edges, each the entire design space of path planning, speed planning, with a constant (and different) speed. and platooning configuration. Our formulation is built upon Other related studies include [29], [35]–[38]. [35] formulates an elegant problem structure that significantly simplifies the a Markov decision process to optimize coordination of platooning optimization. We leverage the formulation to prove of vehicle platooning at highway junctions. [36] presents a new platoon formation strategy that optimizes the platooning that the two-truck platooning problem is weakly NP-hard number and configuration of heterogeneous vehicles in a single and admits a Fully Polynomial Time Approximation Scheme route. [37] proposes a macroscopic model to analyze the (FPTAS). It guarantees to return an approximate and feasible probability distribution of the length of platoons, for a given solution, whose fuel consumption is no larger than $(1 + \epsilon)$ traffic demand. [38] develops an analytical model to investigate times the minimum for any $\epsilon > 0$, with a time complexity the impacts of platoon size. [29] develops a heuristic iterative polynomial in the size of the transportation network and procedure to optimize the routes and departure schedules for. These results are in sharp contrast to the general multi-truck platooning problem that is APX-hard and repels any the general k-platooning problem under deadline constraints, FPTAS [20], [21], suggesting that the two problems are without speed planning involved. The heuristic iterative procedure may not converge or have performance guarantee. fundamentally different. In comparison, in this paper, we explore the full design space, to While the FPTAS works effectively for small-/medium-scale jointly optimize continuous speed planning, path planning, and problem instances, it may still incur excessive running platooning configuration, for a popular two-truck platooning time for large-scale problem instances in practice. We thus problem. The iterative procedure in our scheme is a dual-develop an efficient dual-based algorithm for national-scale subgradient algorithm that guarantees to converge to the optimal problem instances in Sec. III. It is an iterative algorithm optimal, and we provide a posterior performance bound that always converges. Further, we prove that each iteration it. We refer readers to [39] for a comprehensive overview of only incurs polynomial time complexity, albeit it requires studies related to coordinated platooning optimization. solving an integer linear programming problem optimally. We

characterize a condition under which the algorithm generates an optimal solution and derive a posterior performance bound. In Sec. IV, we carry out extensive numerical experiments using real-world traces over the U.S. national highway system. The results show that our algorithm obtains close-to-minimum fuel consumption for the two-truck platooning problem and achieves up to 24% fuel saving as compared to baseline alternatives adapted from state-of-the-art schemes. We conclude and discuss future work in Sec. VI.

II. MODEL AND PROBLEM FORMULATION

A. System Model

We model a national highway network as a directed graph $G = (V; E)$. Each edge $e \in E$ represents a road segment with homogeneous grade, surface type, and environmental conditions. Each node $v \in V$ denotes an intersection or connecting point of adjacent road segments. We denote $n_e, |jV|$ as the number of nodes and $m_e, |jE|$ as the number of edges. For each $e \in E$, we use $D_e > 0$ to denote its length and $r_e^l > 0$ (resp. r_e^u) to denote the minimum (resp. maximum) driving speed.

There are mainly three types of truck fuel consumption models [45]: (i) first-principle white-box models, e.g., the kinematics model based on the Newton's second law [23], [24], [46]–[50]; (ii) black-box models based on fitting a mathematical function to real-world data, e.g., [51]–[58]; (iii) grey-box models that lie between the two models, e.g., [59]. White-box models are developed by using domain knowledge in physical and chemical processes relevant to fuel consumption. It is most accurate and allows one to investigate tuning internal physical/chemical processes for higher fuel efficiency. However, the white-box model usually has a large number of parameters and can be expensive to obtain. On the contrary, the black-box model is less accurate than the white-box model, does not allow internal-process tuning, but has fewer parameters to determine.

For our purpose of minimizing fuel consumption and similar to existing eco-routing studies, we only need to model the (external) speed to fuel-consumption relationship, instead of the whole physical model with internal process tuning. As such, black-box models suffice, and they are easier to construct than white-box models. Indeed, the particular black-box model used in our simulation only has four parameters, and the model predicts the fuel consumption pretty well [60, Fig. 5].

To proceed, we define function $f_e(r_e) : [r_e^l; r_e^u] \rightarrow \mathbb{R}^+$ as the (instantaneous) fuel consumption rate of the truck traveling over e at speed r_e . It is well understood that $f_e(r_e)$ can be modeled as a strictly convex function over the speed range $[r_e^l; r_e^u]$; see e.g., [60], [61] with justifications. Many existing works [62]–[69] also use polynomial functions to model the fuel consumption rate $f_e(\cdot)$. Following the practice, in this paper, we assume that $f_e(\cdot)$ is a strictly

convex polynomial function. As a result, according to Jensen's inequality, driving at a constant speed is most fuel-economic inside a road segment. Thus, without loss of optimality, it suffices to consider trucks traveling at constant speeds over individual road segments [60], [61]. The fuel consumption due to acceleration and deceleration between consecutive road segments is negligible as compared to that of driving over individual segments. For ease of discussion, we further define the fuel consumption function for a truck to traverse e as

$$c_e(t_e), \quad t_e \in f_e(D_e=t_e); 8t_e \in 2 [t_e^l; t_e^u]; \quad (1)$$

where t_e is the travel time of the truck and $t_e^l, D_e=r_e^l$ and $t_e^u, D_e=r_e^u$ are the minimum and maximum travel duration, respectively. We note that $c_e(\cdot)$ is strictly convex over $[t_e^l; t_e^u]$ as its second-order derivative $c_e''(t_e) = f_e''(D_e=t_e) D_e^2 = t_e^3$ is positive. The function has taken into account the influence of road grade, surface type, truck weight, and environment conditions. We define the platooning fuel consumption function when the two trucks platoon on road segments e as

$$c_e^p(t_e) = 2(1 - \alpha) c_e(t_e); \quad (2)$$

where $\alpha \in (0; 1)$ is the average fuel-saving ratio and it usually takes value around 0.1 [20], [25], [26]. Thus when the two trucks platoon at the same speed, they jointly save a fraction of total fuel as compared to driving separately. Note that this model focuses on the total fuel saving; the leading and following trucks can still have different fuel saving performance. Similar to [20], [26], we assume constant fuel-saving rate ratio. More sophisticated fuel models can be derived from Newton's second law, e.g., those in [5], [21], [23], and the fuel-saving rate can be speed-dependent [23]. For these model, it can be verified that the corresponding platooning fuel consumption function is still convex and our approach can be applied directly.

We consider the scenario of two-truck platooning with deadlines. Each truck is associated with a task, denoted by $(s_i; d_i; i_s; i_s; i_d; i_d), i = 1; 2$. Here s_i and d_i are origin and destination for truck i , respectively. The pickup window $[i_s; i_s]$ denotes the time period for pickup at s_i , and the delivery window $[i_d; i_d]$ represents the time period for delivery at d_i . We define the traveling time window of truck $(i = 1; 2)$ as $T_s^i; T_d^i$, where T_s^i, i_s and T_d^i, i_d are the earliest departure time and the latest arrival time, respectively.

B. To Platoon or not to Platoon

It may not always be feasible for the two trucks to platoon without missing deadlines. Even if it is, it may not necessarily save fuel when compared to driving separately. Given a general two-truck platooning instance, we follow the procedure shown in Fig. 1 to obtain a fuel-minimizing solution. We first compute the optimized total fuel consumption of two trucks driving separately, by applying the single-truck

¹This paper considers the setting where the two trucks have identical fuel consumption rate functions. For example, they are the same model and carry the same truckload weight. Our analysis and solution can also be extended to the case with heterogeneous fuel consumption rate functions.

²The speed transition spans over only several hundred feet [70], while trucks travel on a road segment for several miles or longer (3.1 miles on average in the US national highway network as shown in Sec. IV). It is reported in [61] that the fuel usage due to acceleration and deceleration between consecutive road segments is less than 5% of the total fuel consumption.

Fig. 1: The flow chart of our overall solution.

Fig. 2: An illustration of our two-truck platooning problem decomposition.

algorithm [60] to separately optimize the path and speed planning of individual trucks. Next, we apply the polynomial time Algorithm 2 proposed in Appendix VII-D of our technical report [71] to check the feasibility of two-truck platooning under individual deadlines. If infeasible, the two trucks can only drive separately. We output the separate-driving solutions computed in the first step. Otherwise, we solve the feasible two-truck platooning problem with our proposed algorithm to be discussed later. Afterward, we compare the platooning solution and the separate-driving solution and output the one with lower fuel consumption.

With the above understanding, in the rest of the paper, we focus on the feasible two-truck platooning instances.

C. An Elegant Problem Structure and Insights

The conventional formulation of the multi-truck platooning problem introduces a large number of binary variables, each indicating whether a group of trucks platoon over a road segment or not. The formulated problem has a combinatorial structure and is challenging to solve. Indeed, it is APX-hard³ and admits no FPTAS. Interestingly, the following lemma reveals an elegant structure of the feasible two-truck platooning problem, which allows us to significantly simplify the problem formulation.

Lemma 1. For any feasible two-truck platooning instance, there exists an optimal solution in which the two trucks platoon only once.

Proof. See Appendix VII-B of our technical report [71]. \square

Lemma 1 covers the observation in [26] as a special case without deadline constraints and speed planning. It implies

³A problem is APX-hard if it is NP-hard and cannot be approximated within an arbitrary constant factor $(\epsilon > 1)$ in polynomial time unless $P = NP$.

⁴In particular, the authors in [26] show that the multi-truck platooning problem can be reduced to the Steiner tree problem, which is known to be APX-hard and no FPTAS exists [19], [72].

that without loss of optimality, we can focus on solutions in which the two trucks form a platoon only once. This structure significantly reduces the number of platooning configurations to consider, leading to a new formulation that admits efficient algorithms. In particular, applying Lemma 1, we divide the two-truck platooning problem into merging and splitting points selection problem and the following five sub-problems:

- 1) pre-platooning path and speed planning for truck 1;
- 2) pre-platooning path and speed planning for truck 2;
- 3) in-platooning path and speed planning for truck 1 and 2;
- 4) post-platooning path and speed planning for truck 1;
- 5) post-platooning path and speed planning for truck 2.

We denote the five decomposed paths by sub-path 1/2/3/4/5, respectively; see Fig. 2 for an illustration. For ease of presentation, we denote the set \mathcal{N}_k . We introduce binary variables $x_e^i; e \in E; i \in \{1, \dots, 5\}$ for path planning, where

$$x_e^i = \begin{cases} 1; & \text{if edge } e \text{ is selected by the sub-path } i \\ 0; & \text{otherwise.} \end{cases}$$

We also introduce non-negative variables t_e^i to denote the travel time of the (corresponding) trucks over edge e on the sub-path i . Furthermore, we use binary variables s_v and z_v to indicate whether the two trucks start and finish platooning at node $v \in V$, respectively.

D. Problem Formulation

For simplicity of presentation, we denote $\mathbf{x} = (x_e^i; i \in \{1, \dots, 5\}; e \in E)$, $\mathbf{y} = (y_v; v \in V)$, $\mathbf{z} = (z_v; v \in V)$, $\mathbf{t} = (t_e^i; i \in \{1, \dots, 5\}; e \in E)$, and $\mathbf{T} = (t_e^i; i \in \{1, \dots, 5\}; e \in E)$. We denote \mathcal{P} as the feasible set of $(\mathbf{x}; \mathbf{y}; \mathbf{z})$ as follows:

$$\begin{aligned} \mathcal{P} = \{ (\mathbf{x}; \mathbf{y}; \mathbf{z}) \mid & x_e^i, y_v, z_v \in \{0, 1\}; e \in E; i \in \{1, \dots, 5\}; v \in V; \\ & \sum_{e \in \text{Out}(v)} x_e^1 = 1_{v=s_1}, \quad y_v; \forall v \in V; \\ & \sum_{e \in \text{Out}(v)} x_e^2 = 1_{v=s_2}, \quad y_v; \forall v \in V; \\ & \sum_{e \in \text{Out}(v)} x_e^3 = y_v, \quad z_v; \forall v \in V; \\ & \sum_{e \in \text{Out}(v)} x_e^4 = z_v, \quad 1_{v=d_1}; \forall v \in V; \\ & \sum_{e \in \text{Out}(v)} x_e^5 = z_v, \quad 1_{v=d_2}; \forall v \in V; \\ & \sum_{v \in V} y_v = 1; \quad \sum_{v \in V} z_v = 1; \end{aligned} \quad (3)$$

where $\text{Out}(v)$ and $\text{In}(v)$ are the set of outgoing edges and incoming edges of node v , respectively. 1_{fg} is an indicator variable taking value 1 if the statement is true and 0 otherwise. The first five constraints in \mathcal{P} are the flow conservation requirements for the 5 sub-paths. The constraints y_v and z_v restrict the solution to platoon only once.

We now model the deadline constraints and speed planning. The speed planning is done by adjusting the travel time for individual edges. To increase fuel efficiency, we consider departure coordination where trucks can strategically wait at the origins to properly align the schedule to form an efficient platoon later. Consequently, the two trucks start to platoon from the merging point at time

$$\max \left(T_s^1 + \sum_{e \in E} t_e^1 x_e^1, T_s^2 + \sum_{e \in E} t_e^2 x_e^2 \right) : \quad (4)$$

The two expressions in the bracket are the arrival times at the merging point of the two trucks, without departure coordination. If one term in the bracket is smaller than the other, the corresponding truck waits at the origin or a nearby rest area to synchronize the arrival time at the merging point. Without loss of generality, we ignore the cost of strategic waiting at the origin. It can be incorporated into our model by adding a dummy waiting edge at the origin with a designated cost function. To simplify the formulation later, we define

$$1(t, \mathbf{x}) = T_s^1 + \sum_{e \in E} t_e^1 x_e^1 + \sum_{e \in E} t_e^3 x_e^3 + \sum_{e \in E} t_e^4 x_e^4 - T_d^1; \quad (5a)$$

$$2(t, \mathbf{x}) = T_s^2 + \sum_{e \in E} t_e^2 x_e^2 + \sum_{e \in E} t_e^3 x_e^3 + \sum_{e \in E} t_e^4 x_e^4 - T_d^1; \quad (5b)$$

$$3(t, \mathbf{x}) = T_s^1 + \sum_{e \in E} t_e^1 x_e^1 + \sum_{e \in E} t_e^3 x_e^3 + \sum_{e \in E} t_e^5 x_e^5 - T_d^2; \quad (5c)$$

$$4(t, \mathbf{x}) = T_s^2 + \sum_{e \in E} t_e^2 x_e^2 + \sum_{e \in E} t_e^3 x_e^3 + \sum_{e \in E} t_e^5 x_e^5 - T_d^2; \quad (5d)$$

It is straightforward to verify that truck 1 meets its deadline if and only if $1(t, \mathbf{x}) \leq 0$ and $2(t, \mathbf{x}) \leq 0$. Similarly, truck 2 meets deadline if and only if $3(t, \mathbf{x}) \leq 0$ and $4(t, \mathbf{x}) \leq 0$.

To this end, we formulate the feasible two-truck platooning problem as follows:

$$\min \sum_{i \in \{2, 3, 5\}} \sum_{e \in E} c_e(t_e^i) x_e^i + \sum_{e \in E} c_e^0(t_e^3) x_e^3 \quad (6)$$

$$\text{s.t.} \quad j(t, \mathbf{x}) \leq 0; j \in \{2, 4\}; \quad (7)$$

$$\text{var.} \quad (\mathbf{x}; \mathbf{y}; \mathbf{z}) \in \mathcal{P}; \mathbf{t} \in \mathcal{T}$$

The objective in Eq: (6) is the total fuel consumption of the two trucks. Eq: (7) describes their deadline constraints.

We restrict the variables so that the two trucks platoon only once. Compared to the conventional formulation, our formulation in Eq: (7) leverages Lem. 1 to simplify the optimization of platooning configuration, by only considering solutions that platoon only once. Further, the formulation only involves four deadline constraints. We show in Sec. III that this structure significantly simplifies the design of the dual-based algorithm.

E. Hardness

It is not surprising that the two-truck platooning problem is challenging to solve due to its combinatorial structure and multiple deadline constraints. Meanwhile, it is easier than the general multi-truck counterpart.

Theorem 1. The two-truck platooning problem in Eq: (6) is NP-hard, but only in the weak sense. In particular, there is a Fully Polynomial Time Approximation Scheme (FPTAS) for the two-truck platooning problem that achieves $(1 + \epsilon)$ approximation ratio ($\epsilon > 0$) with a time complexity of $O(N^{6/\epsilon^4})$.

Proof. See Appendix VII-C of our technical report [71]. \square

Thm. 1 and its proof show that the two-truck platooning problem is weakly NP-hard and admits an FPTAS. This is in sharp contrast to the general multi-truck platooning problem (even without speed planning), which is APX-hard and repels any FPTAS. Consequently, there exists a pseudo-polynomial time algorithm to solve the two-truck platooning problem exactly. Further, one can trade optimality loss for running time by using the FPTAS derived in the proof. It guarantees to return an approximate and feasible solution to the two-truck platooning problem, whose fuel consumption is within a ratio of $(1 + \epsilon)$ to the optimal for any $\epsilon > 0$, with a time complexity of $O(N^{6/\epsilon^4})$. The FPTAS allows us to solve small-scale two-truck platooning instances efficiently. Meanwhile, it may still incur excessive running time for solving large-scale instances. In the next section, we introduce a dual-subgradient algorithm for solving large-/national- scale instances.

III. AN EFFICIENT DUAL-SUBGRADIENT ALGORITHM

In this section, we design an efficient dual-subgradient algorithm for solving a partially-relaxed dual problem of the two-truck platooning problem, in which only a subset of the constraints are relaxed. While a fully-relaxed dual problem is always convex, the partially-relaxed one is not. Indeed, it is still a combinatorial problem and can be challenging to solve. As a key contribution, we explore the problem structure to show that it can be solved optimally in polynomial time by a dual-subgradient algorithm. We derive a posterior bound on the optimality gap when a feasible primal solution is recovered from the dual optimal solution. We resort to the separate-driving solution when no feasible primal solution is recovered.

A. The Dual of the Two-truck Platooning Problem

We relax the deadline constraints in Eq: (7) and obtain the following Lagrangian function

$$\mathcal{L}(\mathbf{x}; \mathbf{y}; \mathbf{z}; \mathbf{t}; \tilde{\mathbf{t}}) = \sum_{i \in \{2, 3, 5\}} \sum_{e \in E} c_e(t_e^i) x_e^i + \sum_{e \in E} c_e^0(t_e^3) x_e^3 + \sum_{i \in \{2, 4\}} \lambda_i(t, \mathbf{x});$$

where \tilde{t}_e , $(\tilde{x}_1; \tilde{x}_2; \tilde{x}_3; \tilde{x}_4)$ and \tilde{t}_e is the dual variable associated with the constraint $(t_e; x) \leq 0$. The corresponding dual problem is given by

$$\max_{\tilde{t}_e \geq 0} D(\tilde{t}_e), \max_{\tilde{t}_e \geq 0} \min_{(x,y,z) \in 2P} L(x,y,z;t; \tilde{t}_e) : (8)$$

Our idea is to solve this partially-relaxed dual problem optimally and recover the corresponding primal solutions. At the first glance, the inner problem seems challenging as it is still combinatorial (due to the structure of the feasible set). But to proceed, we re-express $D(\tilde{t}_e)$ as follows:

$$D(\tilde{t}_e) = h(\tilde{t}_e) + \min_{(x,y,z) \in 2P} \sum_{i \in N_s} \sum_{e \in E} x_e^i \underbrace{\min_{\substack{t_e^i \in [t_e^l, t_e^u] \\ z}} g_e^i(\tilde{t}_e; t_e^i)}_{\text{speed optimization}} ; \quad (9)$$

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where $h(\tilde{t}_e) = \sum_{i=1}^4 T_s^i T_d^1 + \sum_{i=2}^4 T_s^i T_d^1 + \sum_{i=3}^4 T_s^i T_d^2 + \sum_{i=4}^4 T_s^i T_d^2$ is a linear function and $g_e^i(\tilde{t}_e; t_e^i)$ is a generalized edge cost function given by, for $i \in E$,

$$g_e^i(\tilde{t}_e; t_e^i) = \begin{cases} c_e t_e^1 + (\alpha_1 + \alpha_3) t_e^1, & i = 1; \\ c_e t_e^2 + (\alpha_2 + \alpha_4) t_e^2, & i = 2; \\ c_e t_e^3 + \sum_{j \in N_d} \alpha_j t_e^3, & i = 3; \\ c_e t_e^4 + (\alpha_1 + \alpha_2) t_e^4, & i = 4; \\ c_e t_e^5 + (\alpha_3 + \alpha_4) t_e^5, & i = 5; \end{cases}$$

The function $g_e^i(\tilde{t}_e; t_e^i)$, for all $i \in N_s$ and $e \in E$, is convex in t_e^i as the sum of a convex function and a linear function. As such, the speed optimization module in Eq. (9), one for each edge, can be solved in polynomial time. For all $i \in N_s$ and $e \in E$, define $w_e^i(\tilde{t}_e) = \min_{t_e^i \in [t_e^l, t_e^u]} g_e^i(\tilde{t}_e; t_e^i)$ and $t_e^i(\tilde{t}_e)$ as the optimal objective value and a corresponding optimal solution respectively. We further define $\tilde{x}(\tilde{t}_e) = (t_e^i(\tilde{t}_e))_{i \in N_s; e \in E}$. After obtaining $w_e^i(\tilde{t}_e)$ for all $i \in N_s$ and $e \in E$, the path and platooning optimization module in Eq. (9) amounts to a combinatorial integer linear programming (ILP) problem:

$$\min_{(x,y,z) \in 2P} \sum_{i \in N_s} \sum_{e \in E} w_e^i(\tilde{t}_e) x_e^i : (10)$$

In general, ILP problems can be difficult to solve. General approaches such as the branch-and-bound method incur high computational complexity and do not scale well to large problem instances. As a key contribution, we show that one can relax binary decision variables x, y, z into continuous ones in $[0, 1]$ and solve the resulting linear programming (LP) problem without loss of optimality. Recall that M is the number of edges.

Theorem 2. There is no integrality gap between the ILP problem in Eq. (10) and the corresponding relaxed LP problem. Furthermore, the optimal solution to the former can be recovered from that to the latter in $\mathcal{O}(M^2)$ time.

Proof. The idea is to show every optimal solution of the relaxed LP problem is a convex combination of integer platooning solutions in \mathcal{P} . See Appendix VII-E of our technical

report [71] for the proof. \square

Thm. 2 is an important result. It says we can obtain the optimal solution of the path and platooning optimization problem in Eq. (10), by solving its relaxed LP problem, significantly reducing the computational complexity. Denote the optimal solution of the ILP problem in Eq. (10) as $(x^*(\tilde{t}_e); y^*(\tilde{t}_e); z^*(\tilde{t}_e))$.

To this point, it is straightforward to verify that $(x^*(\tilde{t}_e); y^*(\tilde{t}_e); z^*(\tilde{t}_e))$ is a subgradient of $D(\tilde{t}_e)$ with respect to \tilde{t}_e , $i \in N_d$. From the discussion above, it is clear that both $\tilde{t}_e(\tilde{t}_e)$ and $x^*(\tilde{t}_e)$, hence $(x^*(\tilde{t}_e); y^*(\tilde{t}_e); z^*(\tilde{t}_e))$, can be computed in polynomial time. This allows us to develop an efficient dual-subgradient algorithm for solving the problem in Eq. (8).

B. The Dual-Subgradient Ascent Algorithm

The dual-subgradient algorithm is an iterative algorithm for updating the dual variables towards optimality. Specifically, given $\tilde{t}_e[k]$ as the dual variables after the k th iteration (initially, $\tilde{t}_e[0]$ takes arbitrary non-negative values), the algorithm updates the dual variables as follows: for $i \in N_d$,

$$\tilde{t}_e^i[k+1] = \tilde{t}_e^i[k] + \alpha_k \left(\frac{h(\tilde{t}_e[k])}{\tilde{t}_e^i[k]} - \tilde{t}_e^i[k] \right); \quad (11)$$

where $\tilde{t}_e^i[k]$ and $x^i(\tilde{t}_e[k])$ can be computed in polynomial time by solving the speed optimization and path and platooning optimization modules in Eq. (9), respectively.

$\alpha_k > 0$ is the step size and function $[]^+$ is defined as

$$[]^+ = \begin{cases} ; & \text{if } > 0; \\ \max\{ ; 0\}; & \text{otherwise.} \end{cases}$$

The pseudo-code of the dual-subgradient algorithm is presented in Alg. 1. Intuitively, the dual variables can be interpreted as the delay price. The dual function can be interpreted as a generalized cost involving both the delay and fuel expenses. Higher delay prices lead to shorter truck traveling times, which help to capture deadlines. Meanwhile, higher delay prices also result in higher fuel expense, as seen from the structure of the cost function over individual edges, $g_e^i(\tilde{t}_e; t_e^i)$.

Alg. 1 can thus be understood as adjusting the dual variables according to the subgradients, towards the condition of meeting the deadlines and minimizing the fuel consumption. If Alg. 1 returns at line 7 and all the deadline constraints are satisfied then the corresponding primal solutions are also feasible. Otherwise Alg. 1 returns at line 17. If some deadlines are not met, we perform primal recovery by fixing the returned path planning and platooning solution $(x^*(\tilde{t}_e(K)); y^*(\tilde{t}_e(K)); z^*(\tilde{t}_e(K)))$ and re-optimize the time spent on individual edges. From the simulation results reported in Sec. IV, we observe that such a procedure can recover primary feasible solutions for most instances, with an average

⁵We note that while the dual-subgradient algorithm converges to the dual optimal (as discussed in the next subsection), the deadline constraints may not be satisfied. Indeed, if all the deadline constraints are satisfied at the dual optimal solution, the duality gap is zero, and the dual optimal value is also primal optimal. In general, this may not happen, and the duality gap is nonzero.

Algorithm 1: A Dual subgradient algorithm for optimizing two-truck platooning with deadlines.

```

1 Set  $K, k = 1$ , and  $i[0] = 0; \delta_i \geq 2 N_4$ 
2 Set  $k^u$  as NULL
3 while  $k \leq K$  do
4   Compute  $t^{\sim}[k], x^{\sim}[k], y^{\sim}[k], z^{\sim}[k]$  by solving speed optimization, path and platooning optimization modules Eq: (9).
5   Compute  $i^{\sim}[k]$ ;  $x^{\sim}[k]$  by Eq: (5).
6   if  $i^{\sim}[k] = 0; \delta_i \geq 2 N_4$  then
7     return  $t^{\sim}[k], x^{\sim}[k], y^{\sim}[k], z^{\sim}[k]$ 
8   if  $i^{\sim}[k] > 0; \delta_i \geq 2 N_4$  then
9     Compute the fuel cost  $[k]$  of the solution
10    if  $k^u$  is NULL or  $[k] < [k^u]$  then
11       $k^u = [k]$ 
12    Update  $i[k+1]$  according to Eq: (11),  $\delta_i \geq 2 N_4$ .
13     $k = k + 1$ 
14  if  $k^u$  is not NULL then
15     $k = k^u$ 
16  else
17     $k = K$ 
18 return  $t^{\sim}[k], x^{\sim}[k], y^{\sim}[k], z^{\sim}[k]$ .

```

optimality gap within 1% to the optimal. In the rare situation where the primal recovery fails, we resort to the separate driving solution discussed in Sec. III-B.

C. Performance Analysis

1) Convergence Rate It is known that subgradient algorithms for solving fully-relaxed convex dual problems converge at a rate of $O(1/\sqrt{K})$ [73]. A similar understanding holds for ours for solving the partially-relaxed dual problem in Eq: (8).

Theorem 3. Let D^* be the optimal dual value and D_k be the maximum dual value observed until the k th iteration by running Alg. 1. Then with step sizes $\delta_k = 1/\sqrt{k}$, $i \geq 2 N_4$, there exists a constant $\epsilon > 0$ such that $D_k - D^* \leq \epsilon/k$.

Proof. See Appendix VII-F of our technical report [71]. \square

The constant ϵ depends on the network structure and the deadline constraints. We refer interested readers to the proof for more detailed discussions. Thm. 3 shows that we can achieve $O(1/\sqrt{K})$ convergence rate by using a constant step size. In practice, one may achieve faster convergence by using adaptive step sizes [74].

2) Complexity: The main computational complexity involved in Alg. 1 lies in solving the LP in line 4, which is $O((N + M)^{2.5})$ by the algorithm in [75]. Note that N is the number of nodes, M is the number of edges in the graph, and $O(N + M)$ is the number of variables in the LP. Since the dual-subgradient algorithm converges at a rate of $O(1/\sqrt{K})$, the overall time complexity for Alg. 1 to generate a dual solution within accuracy $\epsilon > 0$ to the optimal is $O((N + M)^{2.5} \sqrt{K})$.

3) Optimality Gap: We provide a posterior bound on the optimality loss when Alg. 1 returns a feasible primal solution. Recall that $t^{\sim}[k]$ consists of the dual variables upon Alg. 1 terminating after K iterations and $i^{\sim}(t^{\sim}[k]; x^{\sim}[k])$ is the corresponding dual subgradient.

Theorem 4. If Alg. 1 returns a feasible solution P , the optimality gap between the cost Bf , denoted as $c(P)$, and the optimal cost, denoted as OPT , is bounded as follows:

$$c(P) - OPT \leq \begin{cases} \delta & \text{if } i^{\sim}() = 0; \delta_i \geq 2 N_4 \text{ and Alg. 1} \\ & \text{returns in line 7} \\ B & \text{otherwise} \end{cases} \quad (12)$$

where $B = \sum_{i=1}^4 P_i [k] i(t^{\sim}[k]; x^{\sim}[k]) \geq 0$.

Proof. The proof is presented in Appendix VII-G of the technical report [71]. \square

According to Thm. 4, if $i^{\sim}() = 0; \delta_i \geq 2 N_4$, i.e., the deadlines for individual trucks are all met exactly, and thus Alg. 1 returns in the line 7, the obtained dual solution is optimal and the corresponding primal solution is feasible and optimal. Otherwise, Alg. 1 returns in the line 17 and we can compute a posterior bound if the corresponding primal solution is feasible. From the simulation results reported in Sec. IV, we observe that for the cases that our algorithm returns feasible primal solution, the computed posterior bounds are within 0.3% to the optimal, suggesting strong empirical performance of the proposed algorithm. Meanwhile, it is clear that the bound holds for each iteration and thus can be used to terminate the Alg. 1 earlier upon reaching a target accuracy threshold. Specially, one can calculate a posterior bound after each iteration and terminate Alg. 1 if the bound is already lower than the target threshold.

IV. PERFORMANCE EVALUATION

We use real-world traces to evaluate the performance of our solutions. We implement our algorithm in python and carry out numerical experiments on a server cluster with 42 pieces of 2.4 GHz – 3.4 GHz Intel/AMD processors, each equipped with 16GB – 96GB memory. We represent an instance of our platooning problem by a tuple $(s_1; d_1; T_s^1; T_d^1; s_2; d_2; T_s^2; T_d^2)$. We follow the flow chart in Fig. 1 to obtain the solution for each instance.

We construct the transportation network and heavy-duty truck network on the US national highway network using the data from the OpenStreetMap (OSM) OpenStreetMap Highway Mapping Project [76]. We focus on the eastern US section with 88,213 nodes and 82,781 road segments. The average length of road segments is 1.5 miles. Our

simulated trucks are both class-8 heavy-duty trucks Kenworth T800, each with a 36-ton full load [77].

Fuel consumption rate function First, we obtain the grade of each road segment by querying the elevations provided by the Elevation Point Query Service [78]. We then use the ADVISOR simulator [79] to collect fuel consumption rate data for each road segment. Finally, we use the curve fitting toolbox in MATLAB to fit the fuel consumption rate function of speed by a 3-order polynomial function. The same polynomial fuel consumption model was also used in other literature of energy-efficient timely truck transportation; see, e.g., [60], [61], [66]–[69], [80].

Platooning fuel consumption For the platooning problem, existing studies on path-only optimization usually assume that the fuel-saving ratio is constant [20], [27]. Studies on speed-only planning often assume the aerodynamic coefficient in the fuel consumption model is reduced at a constant rate [21], [22], [24]. In our simulation, similar to [20], [27], we assume a speed-independent average fuel saving rate for the two trucks, denoted as α in Eq. (2). This is based on the understanding that (i) the fuel saving rates for the leading and trailing vehicle can be different and we are working with their average and (ii) existing studies, e.g., [81], show that the impact of speed on platooning fuel saving rate is minor. Further, our algorithm is applicable without any change as long as the platooning fuel consumption function is convex over the speed on each edge. We use $\alpha = 0.1$ in simulations.

Origin-destination pairs There are few platooning traces available in the public domain. We thus generate synthetic platooning instances from real-world statistics as follows. We first obtain the real-world freight flow statistics in the US from Freight Analysis Framework (FAF) [82]. For each origin-destination pair of the recorded freight flow in FAF, we find the closest highway node and manually set this node as representative truck origin or destination in our simulation. We then randomly sample the first truck origin-destination pair (s_1, d_1) by the freight weight from the origin to the destination. We then calculate the shortest path's distance from s_1 to d_1 . We set a radius lower bound r_l and a radius upper bound r_u . We then set the radius lower bound $r_l = l_1 - r_u$ and the radius upper bound $r_u = l_1 + r_l$. We then sample s_2 (d_2 resp.) from the nodes with distance between r_l and r_u from the s_1 (d_1 resp.), uniformly at random. We set $\alpha = 0.1$ and $\alpha_u = 0.5$. In our simulation, the average distances from s_1 to s_2 and from d_1 to d_2 are 195 miles and 188 miles, respectively.

A. Two Platooning Baselines

We compare the performance of our dual-subgradient algorithm to two baseline methods.

Path planning with platooning (P-Platooning) The first one is based on path planning with the speed of trucks on each edge fixed [20], [27]. The platooning optimization can be formulated as an integer linear programming problem (ILP) and solved by the state-of-the-art solver Gurobi. Specific to our setting, we implement the P-Platooning baseline by setting the speed as the fastest speed and then optimizing the driving paths with platooning consideration.

Speed planning with platooning (S-Platooning) The second one is based on speed planning. Similar to [21], [22], [24], we assume the two trucks travel on their shortest paths. We consider the two trucks may platoon over the overlapping road segments of the two shortest paths. We then optimize speed planning with platooning in consideration subject to deadline constraints, by fixing paths in our formulation in Eq. (6) and using Gurobi to solve the revised problem.

We use the largest dual function value obtained from our dual subgradient algorithm as a non-trivial lower bound to estimate the minimum fuel consumption for evaluating optimality loss.

B. Performance Comparison to Platooning Baselines

Intuitively, by jointly exploiting the design space of path planning, speed planning, and platooning configuration, we can achieve better performance than only optimizing the path planning or the speed planning. We sample the second origin-destination pair within 0.1–0.5 times the shortest path distance of the first distance pair. We define the platooning ratio as the average fraction of platooning in driving distance, and the Shortest Path Overlapping Ratio (S.P.O.R.) as the ratio between the length of overlapping road segments in shortest paths and the total length of two trucks' shortest paths. We then group the results according to the ranging overlapping ratio.

We also introduce a new evaluating metric, platooning fuel saving achieving ratio P defined as the ratio between $\text{OPT}(\text{Path}; \text{Speed}) - \text{Fuel}(\text{Platoon Solution})$ and $\text{OPT}(\text{Path}; \text{Speed}) - \text{OPT}(\text{Path}; \text{Speed}; \text{Platoon})$. $\text{OPT}(\text{Path}; \text{Speed})$ is the optimal fuel cost of path planning and speed planning without platooning, $\text{Fuel}(\text{Platoon Solution})$ is the fuel cost of the platoon solution to be evaluated and $\text{OPT}(\text{Path}; \text{Speed}; \text{Platoon})$ is the optimal fuel cost of path planning and speed planning with platooning. We obtain estimates of the optimal values by using the largest dual value obtained from the dual subgradient algorithm.

We simulate 1495 instances in total. Simulation results in Tab. II show that on average, our solution saves 2.4% (resp. 3%) fuel compared to the P-Platooning (resp. S-Platooning) baselines. We observe that, empirically, our solution's fuel consumption is within less than 0.8% to the lower bound of minimum. The optimality loss of our solution is thus minor.

Furthermore, Fig. 3 shows that when Shortest Path Overlapping Ratio is small, the achieving ratio of S-Platooning is minor or even negative. In contrast, our solution always achieve a high ratio close to one. This result shows that our solution achieves almost the maximum fuel saving offered by platooning.

C. The Benefits of Departure Coordination

In this experiment, we compare the solution with departure coordination to that without departure coordination. Similar to Specifically, we obtain the approximate $\text{OPT}(\text{Path}; \text{Speed})$ by using the dual based method in [60]. We use the largest dual value obtained in our solution, which is within 0.8% to $\text{OPT}(\text{Path}; \text{Speed}; \text{Platoon})$, to approximate $\text{OPT}(\text{Path}; \text{Speed}; \text{Platoon})$.

TABLE II: Fuel saving performance of various platooning solutions, under different shortest path overlapping ratios

Shortest path overlapping ratio	(0.0, 0.2)	(0.2, 0.4)	(0.4, 0.6)	(0.6, 0.8)	(0.8, 1.0)
P-Platooning platooning ratio (%)	45.3	56.6	63.8	76.0	85.2
S-Platooning platooning ratio (%)	3.3	29.6	50.2	69.7	84.7
Our Solution platooning ratio (%)	43.9	56.8	63.4	75.4	85.2
Fuel Saving compared to P-Platooning (%)	22.4	22.8	28.2	25.3	20.2
Fuel Saving compared to S-Platooning (%)	4.3	2.6	1.2	0.8	0.2
Upper bound of our solution's optimality gap (%)	0.2	0.3	0.5	0.5	0.5

Fig. 3: Platooning fuel saving achieving ratio with different Shortest Path Overlapping Ratio. The achieving ratio of S-Platooning decreases to negative, as the overlapping ratio decreases. In contrast, our solution attains an achieving ratio close to one consistently.

Fig. 4: The average fuel cost normalized by the fuel cost of fastest path solution decreases as the delay factor increases. The average waiting time ratio for the waiting truck increases as delay factor increases.

saving, and departure coordination is effective in capitalizing the fuel-saving potential.

the opportunistic driving idea proposed in [80], allowing trucks to wait at the origins can create more favorable platooning opportunity at a later time and thus benefit fuel saving. This improves the fuel economy as long as the two trucks can still catch their deadlines. To obtain the solution without departure coordination, we require the two trucks to arrive at the merging point exactly the same time, i.e., $T_s^1 + t_{e2E}^1 x_e^1 = T_s^2 + t_{e2E}^2 x_e^2$, and the other constraints are the same as in Eq: (7). As for the solution with opportunistic driving, we do not require arrival at the merging point exactly the same time. We use similar dual-based procedures to optimize the path speed planning for the case without departure coordination.

D. Impact of Deadline

We now investigate the impact of deadlines. We define the delay factor to be the ratio between Deadline and Fastest Path's Time Cost. We set the radius ratio lower bound to be $\beta_l = 0:1$ and the radius ratio upper bound to be $\beta_u = 0:5$. Fig. 5 gives the fuel-saving performance of different algorithms as the delay factor varies. It can be seen that our solution saves a significant amount of fuel as compared to other solutions. It can also be seen that as the delay factor increases, the fuel-saving contributed by speed planning becomes larger. This matches our intuition that a larger delay factor allows a bigger design space and thus larger fuel-saving potential for speed planning.

Simulation results show that, with departure coordination, we save 5% more fuel as compared to the case without departure coordination. We also observe that about 25% of the instances that are feasible with departure coordination become infeasible without departure coordination. Fig. 4 shows that the normalized fuel cost with departure coordination is significantly lower and decreases faster than that without departure coordination. We also compare our solution's performance to S-Platooning in terms of achieving platooning fuel saving potential, we also compare different solutions' approximated platooning fuel saving achieving ratios as shown in Fig. 6. We observe that S-Platooning's achieving ratio is rather low, sometimes even negative, indicating that the solution is worse than the baseline without platooning. In contrast, our solution always achieve fuel saving ratio close to 1.

Fig. 5: The average fuel cost normalized by the fuel cost of fastest path solution of different algorithms with different delay factors. Our solution consistently achieves a ratio close to one, while S-platooning perform badly, even worse than the baseline without platooning.

TABLE III: Comparison of different solutions' fuel saving percentage with different delay factors. The row "Platooning" denotes the Platooning Time Percentage of our solution. It can be seen that there is a significant fraction of platooning time (more than 20%), which shows that platooning indeed contributes to fuel saving.

Delay Factor	1.3	1.5	1.7	1.9
P-Platooning (%)	3.4	3.4	3.4	3.4
S-Platooning (%)	17.9	22.7	25.0	25.9
Our solution (%)	20.5	25.0	27.2	28.0
Platooning (%)	35.5	37.3	38.1	37.8

V. DISCUSSION

A. Computation time

We run all the instances (about 1500 in total) on a server cluster with 42 pieces of 2.4 GHz – 3.4 GHz Intel/AMD processors, each equipped with 16GB – 96 GB memory. The total solving time is about 6 hours, less than 15 seconds for one instance even by the most conservative estimate. On a personal computer with Intel(R) Core(TM) i7-10610U processor and 16 GB RAM, it takes 10-20 minutes to solve one instance.

In Sec. III-C, we analyze the time complexity of our proposed algorithm, which is $O((N + M)^{2.5})$ where N is the number of nodes and M is the number of edges in the graph. In simulation, we observe that a large fraction of the computation time is used in obtaining the dual value by solving the relaxed problem in line 4 of Alg. 1, which contains two parts. The first is solving a convex optimization problem for each edge. The second is a linear programming with variable dimension proportional to the sum of number of nodes and number of edges. Thus the computation time depends heavily on the size of the graph. As such, one way to reduce the computation time is to prune the nodes and edges that are away from the origins and destinations of the trucks, which are unlikely to appear in the optimal or near-optimal solution.

B. Perfect and imperfect knowledge of traffic conditions

As the first step to optimize fuel consumption for two-truck platooning under individual deadline constraints, we solve the fuel minimization problem under the setting where traffic conditions are static and known. The obtained solution is feasible and close to optimal when the static traffic condition can be accurately estimated. They can also serve as benchmarks for existing or future two-truck platooning solutions with imperfect static traffic information.

Meanwhile, the real-world traffic conditions can also be time-varying. The current formulation does not consider such variable traffic conditions. A viable workaround is to recompute the solution periodically, e.g., once every hour, using the latest traffic information. It is also conceivable to employ the phase-based traffic model in [80] to incorporate variable traffic conditions into our formulation. The obtained results can be benchmarks for evaluating two-truck platooning solutions with imperfect time-varying traffic information.

VI. CONCLUSION AND FUTURE WORK

This paper studies the two-truck platooning problem to minimize the total fuel consumption while meeting individual deadlines by jointly optimizing path planning, speed planning, and platooning configuration. We show that any feasible two-truck platooning problem has an optimal solution in which the two trucks platoon only once. With this understanding, we develop a new formulation and show that the two-truck platooning problem is only weakly NP-hard and admits an FPTAS. This contrasts with the general multi-truck platooning problem, which is known to be APX-hard and thus repels the development of a polynomial-time approximation scheme for solving large-/national- scale problem instances. It is also shown that the proposed algorithm is a weakly FPTAS. We further design a dual-subgradient algorithm for solving large-/national- scale problem instances. It is shown that the proposed algorithm is a weakly FPTAS. Each iteration involves solving a non-trivial integer linear problem optimally, which we prove incurs only polynomial time complexity. We characterize a sufficient condition under which the algorithm generates an optimal solution. We characterize a posterior

bound on the optimality gap when the condition is not met. We use the real-world traces over the US national highway system to demonstrate the performance of our algorithm. As compared to the path-/speed- only baselines adapted from state-of-the-art schemes, our algorithm achieves up to 24% fuel saving (from path, speed planning and platooning), with an average optimality gap no larger than 0.8%. The results demonstrate the effectiveness of our approach. It is an interesting direction to investigate how the approach in this paper can be extended to the general k-truck platooning settings, by for example dividing the k trucks into multiple groups of two and separately applying the approach to each group for fuel consumption minimization.

REFERENCES

- [1] American Truck Association, "ATA american trucking trends," American Trucking Associations, 2020. [Online]. Available: <https://www.trucking.org/news-insights/ata-american-trucking-trends-2020>.
- [2] World Bank. "Gross domestic product 2019." (2019), [Online]. Available: <https://databank.worldbank.org/data/download/GDP.pdf>.
- [3] N. Williams and D. Murray, "An analysis of the operational costs of trucking: 2020 update," 2020. [Online]. Available: <https://truckingresearch.org/wp-content/uploads/2020/11/ATRI-Operational-Costs-of-Trucking-2020.pdf>.
- [4] International Energy Agency, "The Future of Trucks 2017." [Online]. Available: <https://www.oecd-ilibrary.org/content/publication/9789264279452-en>.
- [5] A. Alam, J. Martensson, and K. H. Johansson, "Look-ahead cruise control for heavy duty vehicle platooning," in 16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013), pp. 928–935.
- [6] B. Besselink, V. Turri, S. H. van de Hoef et al., "Cyber-physical control of road freight transport," in Proceedings of the IEEE, vol. 104, 2016, pp. 1128–1141.
- [7] V. Turri, B. Besselink, and K. H. Johansson, "Cooperative look-ahead control for fuel-efficient and safe heavy-duty vehicle platooning," IEEE Transactions on Control Systems Technology, vol. 25, no. 1, pp. 12–28, 2017.
- [8] C. Bonnet and H. Fritz, "Fuel consumption reduction in a platoon: Experimental results with two electronically coupled trucks at close spacing," SAE Technical Paper, SAE International, Aug. 2000.
- [9] M. P. Lammert, A. Duran, J. Diez, K. Burton, and A. Nicholson, "Effect of platooning on fuel consumption of class 8 vehicles over a range of speeds, following distances, and mass," SAE Int. J. Commer. Veh., vol. 7, pp. 626–639, Sep. 2014.
- [10] A. Alam, B. Besselink, V. Turri, J. Martensson, and K. H. Johansson, "Heavy-duty vehicle platooning for sustainable freight transportation: A cooperative method to enhance safety and efficiency," IEEE Control Systems Magazine, vol. 35, no. 6, pp. 34–56, 2015.
- [11] P. Vegendla, T. Sofu, R. Saha, M. Kumar, and L. Hwang, "Investigation of aerodynamic influence on truck platooning," SAE Technical Paper: 2015-01-2895, 2015.
- [12] S. Tsugawa, S. Jeschke, and S. E. Shladover, "A review of truck platooning projects for energy savings," IEEE Transactions on Intelligent Vehicles, vol. 1, no. 1, pp. 68–77, 2016.
- [13] B. van Arem, C. J. G. van Driel, and R. Visser, "The impact of cooperative adaptive cruise control on traffic flow characteristics," IEEE Transactions on Intelligent Transportation Systems, vol. 7, no. 4, pp. 429–436, 2006.
- [14] S. Tsugawa, S. Kato, and K. Aoki, "An automated truck platoon for energy saving," in 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems 2011, pp. 4109–4114.
- [15] M. A. Tunnell, "Estimating truck-related fuel consumption and emissions in maine: A comparative analysis for six-axle, 100,000 pound vehicle configuration," Proc. TRB Annual Meeting 2011.
- [16] Amazon, "Place an order with guaranteed delivery." [Online]. Available: https://www.amazon.com/gp/help/customer/display.html/ref=help_v4_sib?ie=UTF8&nodeId=201117390.
- [17] B. H. Ashby, "Protecting Perishable Foods during Transport by Truck U.S. Department of Agriculture, 2006. [Online]. Available: <https://www.ams.usda.gov/sites/default/files/media/TransportPerishableFoodsbyTruck%5C%5B1%5C%5D.pdf>.
- [18] W. Mallett, "Freight Performance Measurement: Travel Time in Freight-Significant Corridors U.S. Federal Highway Administration, 2006. [Online]. Available: https://ops.fhwa.dot.gov/freight/freightanalysis/perform_meas/fpmtraveltime/index.htm.
- [19] D. E. Drake and S. Hougardy, "On approximation algorithms for the terminal Steiner tree problem," Information Processing Letters, vol. 89, no. 1, pp. 15–18, 2004.
- [20] J. Larson, T. Munson, and V. Sokolov, "Coordinated platoon routing in a metropolitan network," in 2016 Proceedings of the Seventh SIAM Workshop on Combinatorial Scientific Computing 2016, pp. 73–82.
- [21] S. van de Hoef, K. H. Johansson, and D. V. Dimarogonas, "Fuel-optimal centralized coordination of truck platooning based on shortest paths," 2015 American Control Conference (ACC), pp. 3740–3745.
- [22] S. Van De Hoef, K. H. Johansson, and D. V. Dimarogonas, "Coordinating truck platooning by clustering pairwise fuel-optimal plans," in 2015 IEEE 18th International Conference on Intelligent Transportation Systems 2015, pp. 408–415.
- [23] K. Liang, J. Martensson, and K. H. Johansson, "Heavy-duty vehicle platoon formation for fuel efficiency," IEEE Transactions on Intelligent Transportation Systems, vol. 17, no. 4, pp. 1051–1061, 2016.
- [24] S. van de Hoef, K. H. Johansson, and D. V. Dimarogonas, "Fuel-efficient en route formation of truck pla-

- toons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 1, pp. 102–112, 2018.
- [25] J. Larson, C. Kammer, K. Liang, and K. H. Johansson, "Coordinated route optimization for heavy-duty vehicle platoons," in *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, pp. 1196–1202.
- [26] E. Larsson, G. Sennton, and J. Larson, "The vehicle platooning problem: Computational complexity and heuristics," *Transportation Research Part C: Emerging Technologies*, vol. 60, pp. 258–277, 2015.
- [27] F. Luo, J. Larson, and T. Munson, "Coordinated platooning with multiple speeds," *Transportation Research Part C: Emerging Technologies*, vol. 90, pp. 213–225, 2018.
- [28] N. Boysen, D. Briskorn, and S. Schwerdfeger, "The identical-path truck platooning problem," *Transportation Research Part B: Methodological*, vol. 109, pp. 26–39, 2018.
- [29] F. Luo and J. Larson, "A repeated route-then-schedule approach to coordinated vehicle platooning: Algorithms, valid inequalities and computation," *Operations Research*, 2021.
- [30] W. Zhang, E. Jenelius, and X. Ma, "Freight transport platoon coordination and departure time scheduling under travel time uncertainty," *Transportation Research Part E: Logistics and Transportation Review*, vol. 98, pp. 1–23, 2017.
- [31] R. Larsen, J. Rich, and T. K. Rasmussen, "Hub-based truck platooning: Potentials and profitability," *Transportation Research Part E: Logistics and Transportation Review*, vol. 127, pp. 249–264, 2019.
- [32] A. Nourmohammadzadeh and S. Hartmann, "The fuel-efficient platooning of heavy duty vehicles by mathematical programming and genetic algorithm," *Theory and Practice of Natural Computing*, 2016, pp. 46–57.
- [33] A. Nourmohammadzadeh and S. Hartmann, "Fuel-efficient truck platooning by a novel meta-heuristic inspired from ant colony optimisation," *Soft Computing*, vol. 23, pp. 1–14, Sep. 2018.
- [34] A. Nourmohammadzadeh and S. Hartmann, "Fuel-efficient truck platooning with time restrictions and multiple speeds solved by a particle swarm optimisation," in *International Conference on Theory and Practice of Natural Computing 2018*, pp. 188–200.
- [35] X. Xiong, J. Sha, and L. Jin, "Optimizing coordinated vehicle platooning: An analytical approach based on stochastic dynamic programming," *Transportation Research Part B: Methodological*, vol. 150, pp. 482–502, 2021.
- [36] W. J. Lee, S. I. Kwag, and Y. D. Ko, "The optimal eco-friendly platoon formation strategy for a heterogeneous fleet of vehicles," *Transportation Research Part D: Transport and Environment*, vol. 90, p. 102664, 2021.
- [37] M. Sala and F. Soriguera, "Capacity of a freeway lane with platoons of autonomous vehicles mixed with regular traffic," *Transportation Research Part B: Methodological*, vol. 147, pp. 116–131, 2021.
- J. Zhou and F. Zhu, "Analytical analysis of the effect of maximum platoon size of connected and automated vehicles," *Transportation Research Part C: Emerging Technologies*, vol. 122, p. 102882, 2021.
- A. K. Bhoopalani, N. Agatz, and R. Zuidwijk, "Planning of truck platoons: A literature review and directions for future research," *Transportation Research Part B: Methodological*, vol. 107, pp. 212–228, 2018.
- Ricardo, TRL, and TTR, "Heavy vehicle platoons on UK roads," 2014. [Online]. Available: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/637361/truck-platooning-uk-feasibility-study.pdf.
- C. Crane, J. Bridge, and R. Bishop, "Driver assistive truck platooning: Considerations for Florida state agencies," 2018. [Online]. Available: <https://fdotwww.blob.core.windows.net/sitefinity/docs/default-source/content/legislative/documents/datp.pdf?sfvrsn=971162d9>
- J. Roberts, R. Mihelic, and M. Roeth, "Conference report: Two-truck platooning," 2016. [Online]. Available: https://nacfe.org/wp-content/uploads/2018/02/TE-Platooning-CR-FINAL_0.pdf.
- G. Janssen, J. Zwijnenberg, I. Blankers, and J. D. Kruijff, "Truck platooning: Driving the future of transportation," 2015. [Online]. Available: <http://resolver.tudelft.nl/uuid:778397eb-59d3-4d23-9185-511385b91509>.
- J. Andersson, C. Englund, and A. Voronov, "Study of communication needs in interaction between trucks and surrounding traffic in platooning," 2017. [Online]. Available: https://www.drivesweden.net/sites/default/files/content/nal_report_-_40016_submitted.pdf.
- M. Zhou, H. Jin, and W. Wang, "A review of vehicle fuel consumption models to evaluate eco-driving and eco-routing," *Transportation Research Part D: Transport and Environment*, vol. 49, pp. 203–218, 2016.
- L. Cadrón and E. Pucher, "Fuel consumption simulation model of a cng vehicle based on real-world emission measurement," *SAE Technical Paper, Tech. Rep.*, 2007.
- J. B. Heywood, *Internal combustion engine fundamentals*. McGraw-Hill Education, 2018. [Online]. Available: <https://www.accessengineeringlibrary.com/content/book/9781260116106>.
- H. A. Rakha, K. Ahn, K. Moran, B. Saerens, and E. V. den Bulck, "Virginia tech comprehensive power-based fuel consumption model: Model development and testing," *Transportation Research Part D: Transport and Environment*, vol. 16, no. 7, pp. 492–503, 2011.
- J. J. Moskwa and J. K. Hedrick, "Modeling and validation of automotive engines for control algorithm development," *Journal of Dynamic Systems, Measurement, and Control*, 1992.
- G. Guo and Q. Wang, "Fuel-efficient en route speed planning and tracking control of truck platoons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 8, pp. 3091–3103, 2019.
- B. Saerens, H. Rakha, K. Ahn, and E. V. D. Bulck, "Assessment of alternative polynomial fuel consumption models for use in intelligent transportation systems

- applications,” *Journal of Intelligent Transportation Systems*, vol. 17, no. 4, pp. 294–303, 2013.
- [52] E. Hellström, M. Ivarsson, J. Åslund, and L. Nielsen, “Look-ahead control for heavy trucks to minimize trip time and fuel consumption,” *Control Engineering Practice*, vol. 17, no. 2, pp. 245–254, 2009.
- [53] S. Park, H. Rakha, M. Farzaneh, J. Zietsman, and D.-W. Lee, “Development of fuel and emission models for high speed heavy duty trucks, light duty trucks, and light duty vehicles,” in *13th International IEEE Conference on Intelligent Transportation Systems*, 2010, pp. 25–32.
- [54] K. Post, J. H. Kent, J. Tomlin, and N. Carruthers, “Fuel consumption and emission modelling by power demand and a comparison with other models,” *Transportation Research Part A: General*, vol. 18, no. 3, pp. 191–213, 1984.
- [55] W. F. Faris, H. A. Rakha, R. I. Kafafy, M. Idres, and S. Elmoselhy, “Vehicle fuel consumption and emission modelling: An in-depth literature review,” *International Journal of Vehicle Systems Modelling and Testing*, vol. 6, no. 3-4, pp. 318–395, 2011.
- [56] B. Piccoli, K. Han, T. L. Friesz, and T. Yao, “Estimating fuel consumption and emissions via traffic data from mobile sensors,” in *2013 51st Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, 2013, pp. 472–477.
- [57] D. Y. C. Leung and D. J. Williams, “Modelling of motor vehicle fuel consumption and emissions using a power-based model,” in *Urban Air Quality: Measurement, Modelling and Management*, Springer, 2000, pp. 21–29.
- [58] K. Ahn, H. Rakha, A. Trani, and M. V. Aerde, “Estimating vehicle fuel consumption and emissions based on instantaneous speed and acceleration levels,” *Journal of Transportation Engineering*, vol. 128, no. 2, pp. 182–190, 2002.
- [59] L. Pelkmans, P. Debal, T. Hood, G. Hauser, and M. Delgado, “Development of a simulation tool to calculate fuel consumption and emissions of vehicles operating in dynamic conditions,” SAE Technical Paper, Tech. Rep., 2004.
- [60] L. Deng, M. H. Hajiesmaili, M. Chen, and H. Zeng, “Energy-efficient timely transportation of long-haul heavy-duty trucks,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 7, pp. 2099–2113, 2018.
- [61] Q. Liu, H. Zeng, and M. Chen, “Energy-efficient timely truck transportation for geographically-dispersed tasks,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 12, pp. 5148–5159, 2020.
- [62] E. K. Nam and R. Giannelli, “Fuel consumption modeling of conventional and advanced technology vehicles in the physical emission rate estimator (PERE),” *US Environmental Protection Agency*, 2005.
- [63] S. Ardenkani, E. Hauer, and B. Jamei, “Traffic impact models,” *Traffic Flow Theory: A State-of-the Art Report*, vol. 7, pp. 7–24, Jan. 2001.
- [64] F. An and M. Ross, “Model of fuel economy with applications to driving cycles and traffic management,” *Transportation Research Record*, 1993.
- [65] H. Yue, “Mesoscopic fuel consumption and emission modeling,” 2008. [Online]. Available: <http://hdl.handle.net/10919/26695>.
- [66] D. W. Gao, C. Mi, and A. Emadi, “Modeling and simulation of electric and hybrid vehicles,” *Proceedings of the IEEE*, vol. 95, no. 4, pp. 729–745, 2007.
- [67] Y. M. Nie and Q. Li, “An eco-routing model considering microscopic vehicle operating conditions,” *Transportation Research Part B: Methodological*, vol. 55, pp. 154–170, 2013.
- [68] E. D., T. B., and G. Laporte, “A comparative analysis of several vehicle emission models for road freight transportation,” *Transportation Research Part D: Transport and Environment*, 2011.
- [69] A. Lajunen, “Energy consumption and cost-benefit analysis of hybrid and electric city buses,” *Transportation Research Part C: Emerging Technologies*, vol. 38, pp. 1–15, 2014.
- [70] G. Yang, H. Xu, Z. Wang, and Z. Tian, “Truck acceleration behavior study and acceleration lane length recommendations for metered on-ramps,” *International Journal of Transportation Science and Technology*, vol. 5, no. 2, pp. 93–102, 2016.
- [71] W. Xu, T. Cui, and M. Chen, *Optimizing two-truck platooning with deadlines*, 2022. DOI: 10.48550/ARXIV.2210.01889. [Online]. Available: <https://arxiv.org/abs/2210.01889>.
- [72] M. Bern and P. Plassmann, “The Steiner problem with edge lengths 1 and 2,” *Information Processing Letters*, vol. 32, no. 4, pp. 171–176, 1989.
- [73] S. Boyd, L. Xiao, and A. Mutapcic, “Subgradient methods,” *Lecture Notes of EE392o, Stanford University, Autumn Quarter*, 2003. [Online]. Available: https://web.stanford.edu/class/ee392o/subgrad_method.pdf.
- [74] M. Bazaraa and H. D. Sherali, “On the choice of step size in subgradient optimization,” *European Journal of Operational Research*, vol. 7, no. 4, pp. 380–388, 1981.
- [75] P. Vaidya, “Speeding-up linear programming using fast matrix multiplication,” in *Proceedings of the 30th Annual Symposium on Foundations of Computer Science*, ser. SFCS ’89, USA: IEEE Computer Society, 1989, pp. 332–337.
- [76] J. D. Teresco, “The clinched highway mapping (CHM) project.” [Online]. Available: <http://cmap.m-plex.com/>.
- [77] Kenworth, “Kenworth T800 vehicle.” [Online]. Available: <http://www.kenworth.com/trucks/t800>.
- [78] U.S. Geological Survey, “USGS elevation point query service.” [Online]. Available: <http://nationalmap.gov/epqs/>.
- [79] T. Markel, A. Brooker, T. Hendricks, *et al.*, “ADVISOR: A systems analysis tool for advanced vehicle modeling,” *Journal of Power Sources*, vol. 110, pp. 255–266, 2002.
- [80] W. Xu, Q. Liu, M. Chen, and H. Zeng, “Ride the tide of traffic conditions: Opportunistic driving improves energy efficiency of timely truck transportation,” in

